DKSST's

Dr. A. D. Shind College of Engineering, Bhadgaon.

Laboratory Manual

Theory of Machine-I

For

Second Year Students

Department of Mechanical Engineering

INDEX

Experiment No: 1

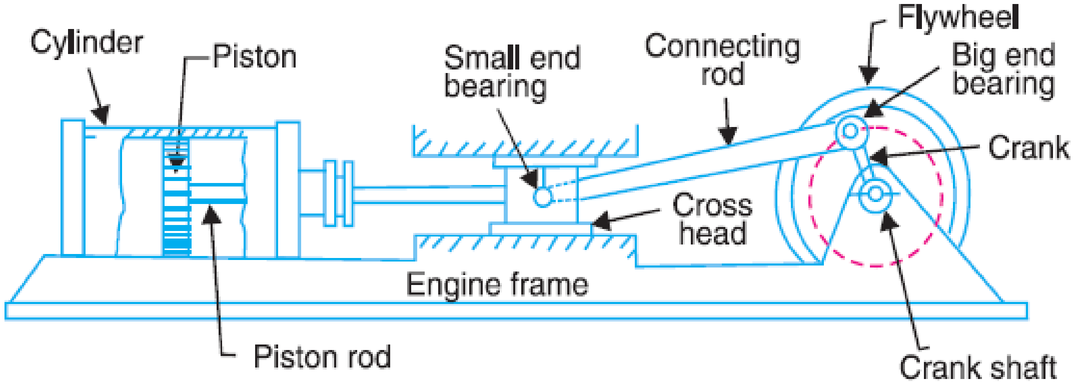
Study of Kinematics and definitions

Introduction:

Machine is a device which receives energy and transforms it into some useful work. A machine consists of a number of parts or bodies. We shall study the mechanisms of the various parts or bodies from which the machine is assembled. This is done by making one of the parts as fixed, and the relative motion of other parts is determined with respect to the fixed part.

Kinematic Link or Element:

Each part of a machine, which moves relative to some other part, is known as a kinematic link (or simply link) or element. A link may consist of several parts, which are rigidlyfastened together, so that they do not move relative to one another. For example, in a reciprocating steam engine, as shown in Fig. piston, piston rod and crosshead constitute one link ; connecting rod with big and small end bearings constitute a second link ; crank, crank shaft and flywheela third link and the cylinder, engine frame and main bearings a fourth link.



A link or element needs not to be a rigid body, but it must be a resistant body. A body is said to be a resistant body if it is capable of transmitting the required forces with negligible deformation. Thus a link should have the following two characteristics:
1. It should have relative motion, and
2. It must be a resistant body.

Types of Links

In order to transmit motion, the driver and the follower may be connected by the following three types of links :

1. Rigid link. A rigid link is one which does not undergo any deformation while transmitting motion. Strictly speaking, rigid links do not exist. However, as the deformation of a connecting rod, crank etc. of a reciprocating steam engine is not appreciable, they can be considered as rigid links.

2. Flexible link. A flexible link is one which is partly deformed in a manner not to affect the

transmission of motion. For example, belts, ropes, chains and wires are flexible links and transmit tensile forces only.

3. Fluid link. A fluid link is one which is formed by having a fluid in a receptacle and the motion is transmitted through the fluid by pressure or compression only, as in the case of hydraulic presses, jacks and brakes.

Structure:

It is an assemblage of a number of resistant bodies (known as members) having no relative motion between them and meant for carrying loads having straining action. A railway bridge, a roof truss, machine frames etc., are the examples of a structure.

Difference between a Machine and a Structure

The following differences between a machine and a structure are important from the subject point of view:

1. The parts of a machine move relative to one another, whereas the members of a structure do not move relative to one another.
2. A machine transforms the available energy into some useful work, whereas in a structure no energy is transformed into useful work.
3. The links of a machine may transmit both power and motion, while the members of a structure transmit forces only.

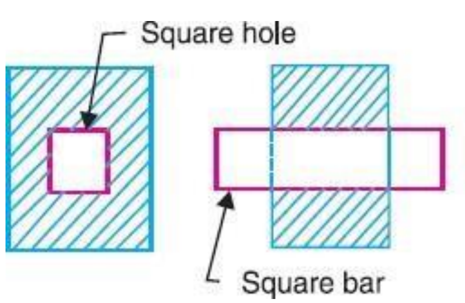
Kinematic Pair

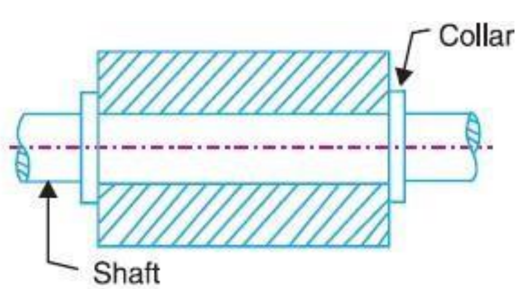
The two links or elements of a machine, when in contact with each other, are said to form a pair. If the relative motion between them is completely or successfully constrained (i.e. in a definite direction), the pair is known as kinematic pair.

Types of Constrained Motions

Following are the three types of constrained motions:

1. **Completely constrained motion.** When the motion between a pair is limited to a definite direction irrespective of the direction of force applied, then the motion is said to be a completely constrained motion. For example, the piston and cylinder (in a steam engine) form a pair and the motion of the piston is limited to a definite direction (i.e. it will only reciprocate) relative to the cylinder irrespective of the direction of motion of the crank, as shown in Fig





2. **Incompletely constrained motion**. When the motion between a pair can take place in more than one direction, then the motion is called an incompletely constrained motion. The change in the direction of impressed force may alter the direction of relative motion between the pair. A circular bar or shaft in a circular hole, as shown in Fig. is an example of an incompletely

constrained motion as it may either rotate or slide in a hole. These both motions have no relationship with the other

3. Successfully constrained motion.

When the motion between the elements, forming a pair,is such that the constrained motion is not completed by itself, but by some other means, then the motion is said to be successfully constrained motion. Consider a shaft in a foot-step bearing as shown in Fig.. The shaft may rotate in a bearing or it may move upwards. This is a case of incompletely constrained motion. But if the load is placed on the shaft to prevent axial upward movement of the shaft, then the motion of the pair is said to be successfully constrained motion. The motion of an I.C. engine valve (these are kept on their seat by a spring) and the piston reciprocating inside an engine cylinder are also the examples of successfully constrained motion.

Classification of Kinematic Pairs

The kinematic pairs may be classified according to the following considerations:

1. **According to the type of relative motion between the elements**. The kinematic pairs according to type of relative motion between the elements may be classified as discussed below:

(a) Sliding pair. When the two elements of a pair are connected in such a way that one can only slide relative to the other, the pair is known as a sliding pair. The piston and cylinder, cross-head and guides of a reciprocating steam engine, ram and its guides in shaper, tail stock on the lathe bed etc. are the examples of a sliding pair. A little consideration will show, that a sliding pair has a completely constrained motion.

(b) Turning pair. When the two elements of a pair are connected in such a way that one can only turn or revolve about a fixed axis of another link, the pair is known as turning pair. A shaft with collars at both ends fitted into a circular hole, the crankshaft in a journal bearing in an engine, lathe spindle supported in head stock, cycle wheels turning over their axles etc. are the examples of a turning pair. A turning pair also has a completely constrained motion.

(c) Rolling pair. When the two elements of a pair are connected in such a way that one rolls over another fixed link, the pair is known as rolling pair. Ball and roller bearings are examples of rolling pair.

(d) Screw pair. When the two elements of a pair are connected in such a way that one element can turn about the other by screw threads, the pair is known as screw pair. The lead screw of a lathe with nut, and bolt with a nut are examples of a screw pair.

(e) Spherical pair. When the two elements of a pair are connected in such a way that one element (with spherical shape) turns or swivels about the other fixed element, the pair formed is called a spherical pair. The ball and socket joint, attachment of a car mirror, pen stand etc., are the examples of a spherical pair.

2. **According to the type of contact between the elements**. The kinematic pairs according to the type of contact between the elements may be classified as discussed below :

(a) Lower pair. When the two elements of a pair have a surface contact when relative motion takes place and the surface of one element slides over the surface of the other, the pair formed is known as lower pair. It will be seen that sliding pairs, turning pairs and screw pairs form lower pairs.

(b) Higher pair. When the two elements of a pair have a line or point contact when relative motion takes place and the motion between the two elements is partly turning and partly sliding,then the pair is known as higher pair. A pair of friction discs, toothed gearing, belt and rope drives, ball and roller bearings and cam and follower are the examples of higher pairs.

3. **According to the type of closure.** The kinematic pairs according to the type of closure between the elements may be classified as discussed below :
(a) Self closed pair. When the two elements of a pair are connected together mechanically in such a way that only required kind of relative motion occurs, it is then known as self closed pair. The lower pairs are self closed pair.
(b) Force - closed pair. When the two elements of a pair are not connected mechanically but are kept in contact by the action of external forces, the pair is said to be a force-closed pair. The cam and follower is an example of force closed pair, as it is kept in contact by the forces exerted by spring and gravity.

Inversion of Mechanism

We have already discussed that when one of links is fixed in a kinematic chain, it is called a mechanism. So we can obtain as many mechanisms as the number of links in a kinematic chain by fixing, in turn, different links in a kinematic chain. This method of obtaining different mechanisms by fixing different links in a kinematic chain, is known as inversion of the mechanism.

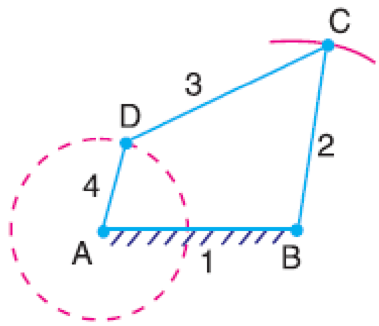
It may be noted that the relative motions between the various links is not changed in any manner through the process of inversion, but their absolute motions (those measured with respect to the fixed link) may be changed drastically

Types of Kinematic Chains

The most important kinematic chains are those which consist of four lower pairs, each pair being a sliding pair or a turning pair. The following three types of kinematic chains with four lower pairs are important from the subject point of view:
1. Four bar chain or quadric cyclic chain,
2. Single slider crank chain, and
3. Double slider crank chain.
These kinematic chains are discussed, in detail, in the following articles.

Four Bar Chain or Quadric Cycle Chain

We have already discussed that the kinematic chain is a combination of four or more kinematic pairs, such that the relative motion between the links or elements is completely constrained.The simplest and the basic kinematic chain is a four bar chain or quadric cycle chain, as shown in Fig. It consists of four links, each of them forms a turning pair at A, B, C and D. The four links may be of different lengths. According to Grashof 's law for a four bar mechanism, the sum of the shortest and longest link lengths should not be greater than the sum of the remaining two link lengths if there is to be continuous relative motion between the two links. A very important consideration in designing a mechanism is to ensure that the input crank makes a complete revolution relative to theother links. The mechanism in which no link makes a complete revolution will not be useful. In a four bar chain, one of the links, in particular the shortest link, will make a complete revolution relative to the other three links, if it satisfies the Grashof 's law. Such a link is known as crank or driver. In Fig. AD (link 4) is a crank. The link BC (link 2) which makes a partial rotation or oscillates is known as lever or rocker or follower and the link CD (link 3) which connects the crank and lever is called connecting rod or coupler. The fixed link AB (link 1) is known as frame of the mechanism.

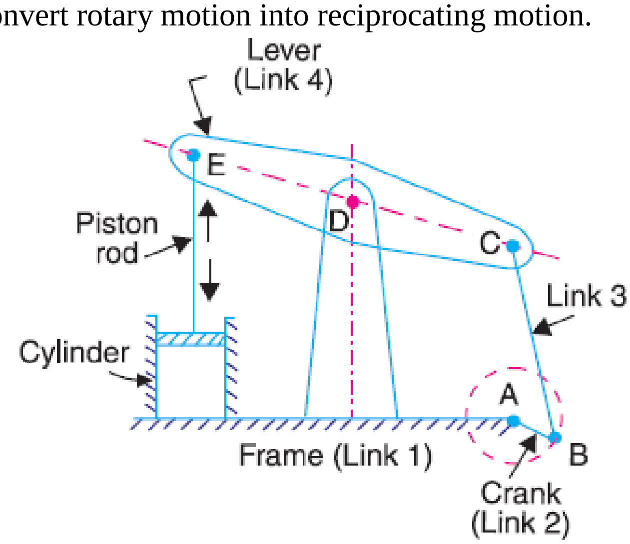


Inversions of Four Bar Chain

Though there are many inversions of the four bar chain, yet the following are important from the subject point of view :

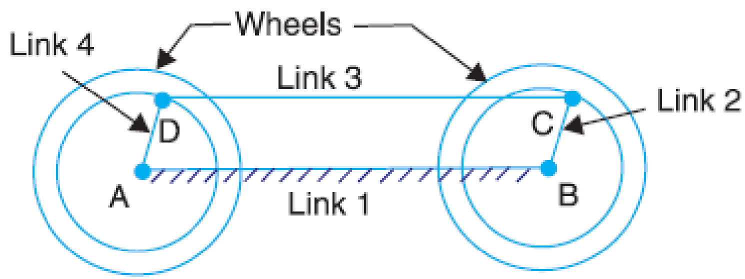
1. Beam engine (crank and lever mechanism)

. A part of the mechanism of a beam engine (also known as crank and lever mechanism) which consists of four links, is shown in Fig. In this mechanism, when the crank rotates about the fixed centre A, the lever oscillates about a fixed centre D. The end E of the lever CDE is connected to a piston rod which reciprocates due to the rotation of the crank. In other words, the purpose of this mechanism is to convert rotary motion into reciprocating motion.



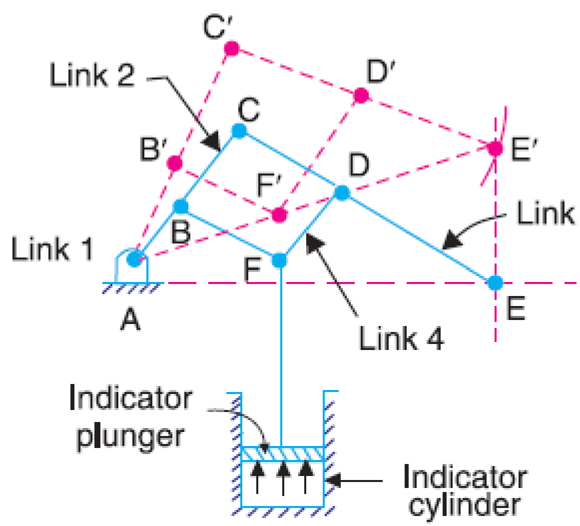
2. Coupling rod of a locomotive (Double crank mechanism).

The mechanism of a coupling rod of a locomotive (also known as double crank mechanism) which consists of four links, is shown in FigIn this mechanism, the links AD and BC (having equal length) act as cranks and are connected to the respective wheels. The link CD acts as a coupling rod and the link AB is fixed in order to maintain a constant centre to centre distance between them. This mechanism is meant for transmitting rotary motion from one wheel to the other wheel.



3. Watt's indicator mechanism (Double lever mechanism).

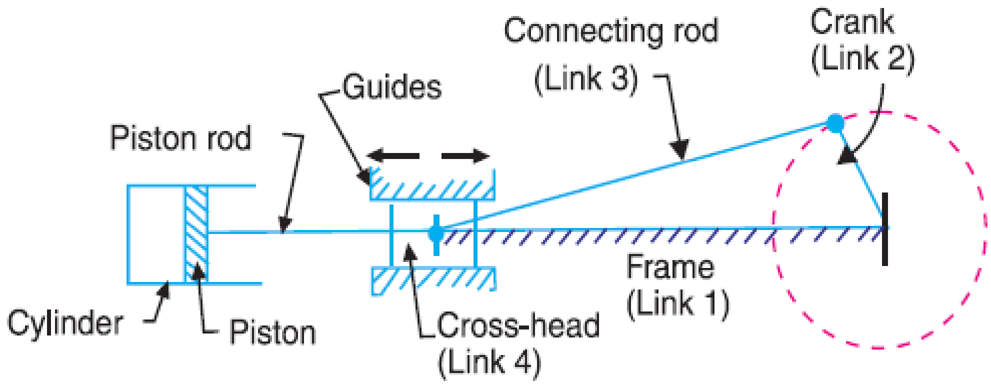
A Watt's indicator mechanism (also known as Watt's straight line mechanism or double lever mechanism) which consists of fourlinks, is shown in Fig. The four links are : fixed linkat A, link AC, link CE and link BFD. It may be noted that BF and FD form one link because these two parts have no relative motion between them. The links CE and BFD act as levers. The displacement of the link BFD is directly proportional to the pressure of gas or steam which acts on the indicator plunger. On any small displacement of the mechanism, the tracing point E at the end of the link CE traces out approximately a straight line.



The initial position of the mechanism is shown in Fig. by full lines whereas the dotted lines show the position of the mechanism when the gas or steam pressure acts on the indicator plunger.

Single Slider Crank Chain

A single slider crank chain is a modification of the basic four bar chain. It consist of one sliding pair and three turning pairs. It is,usually, found in reciprocating steam engine mechanism. This type of mechanism converts rotary motion into reciprocating motion and vice versa. In a single slider crank chain, as shown in Fig, the links 1 and 2, links 2 and 3, and links 3 and 4 form three turning pairs while the links 4 and 1 form a sliding pair.



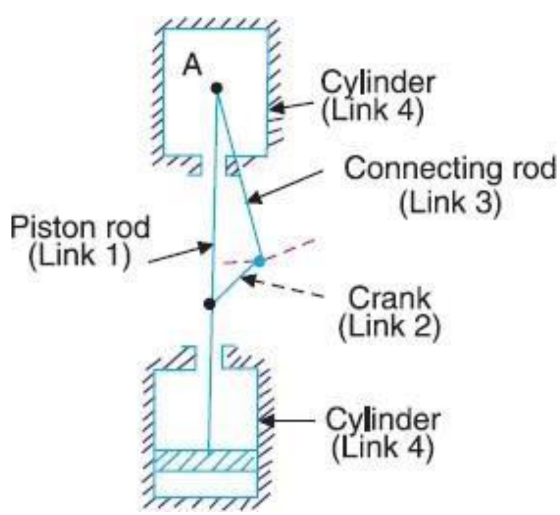
The link 1 corresponds to the frame of the engine, which is fixed. The link 2 corresponds to the crank ; link 3 corresponds to the connecting rod and link 4 corresponds to cross-head. As the crank rotates, the cross-head reciprocates in the guides and thus the piston reciprocates in the cylinder.

Inversions of Single Slider Crank Chain

We have seen in the previous article that a single slider crank chain is a four-link mechanism. We know that by fixing, in turn, different links in a kinematic chain, an inversion is obtained and we can obtain as many mechanisms as the links in a kinematic chain. It is thus obvious, that four inversions of a single slider crank chain are possible. These inversions are found in the following mechanisms.

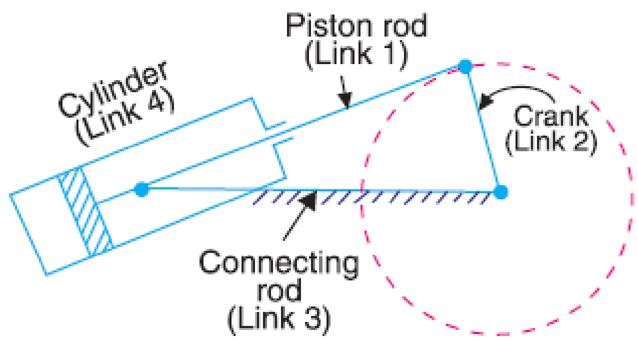
1. Pendulum pump or Bull engine.

In this mechanism, the inversion is obtained by fixing the cylinder or link 4 (i.e. sliding pair), as shown in FigIn this case, when the crank (link 2) rotates,t he connecting rod (link 3) oscillates about a pin pivoted to the fixed link 4 at A and the piston attached to the piston rod (link 1) reciprocates. The duplex pump which is used to supply feed water to boilers have two pistons attached to link 1, as shown in Fig.



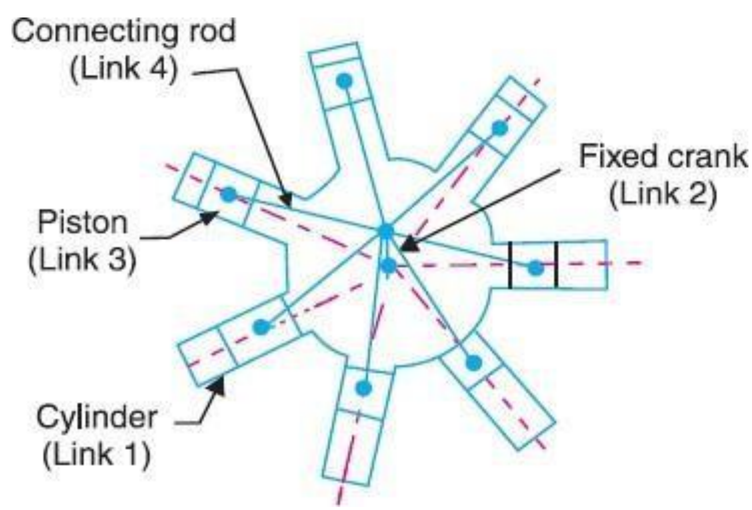
2. Oscillating cylinder engine.

The arrangement of oscillating cylinder engine mechanism, as shown in Fig, is used to convert reciprocating motion into rotary motion. In this mechanism, the link 3 forming the turning pair is fixed. The link 3 corresponds to the connecting rod of a reciprocating steam engine mechanism. When the crank (link 2) rotates, the piston attached to piston rod (link 1) reciprocates and the cylinder (link 4) oscillates about a pin pivoted to the fixed link at A.



Rotary internal combustion engine

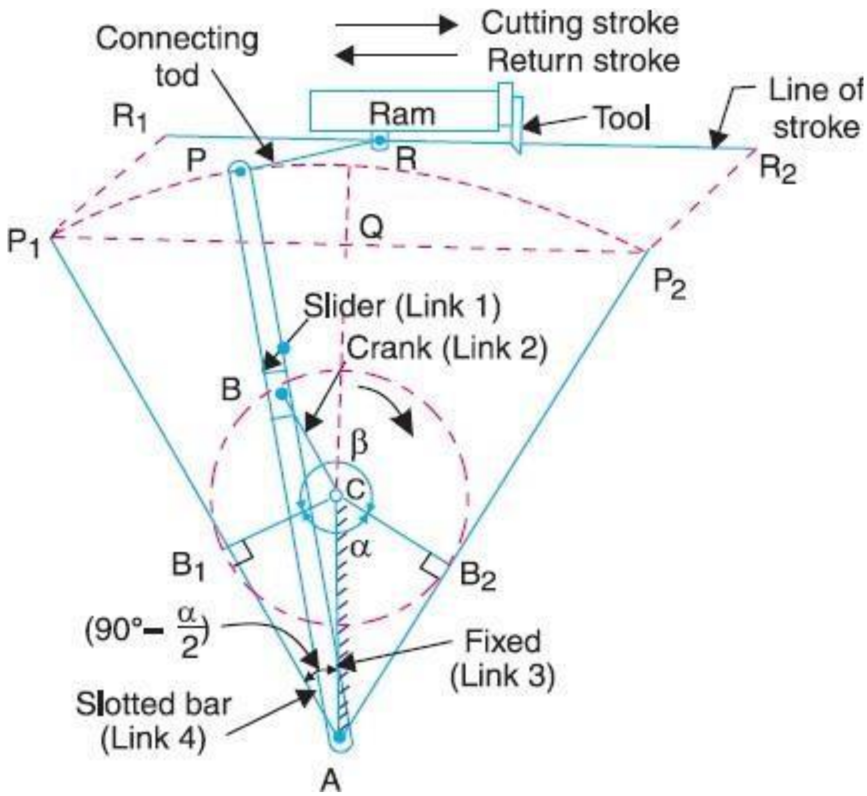
Sometimes back, rotary internal combustion engines were used in aviation. But now-a-days gas turbines are used in its place. It consists of seven cylinders in one plane and all revolves about fixed centre D, as shown in Fig. While the crank (link 2) is fixed. In this mechanism, when the connecting rod (link 4) rotates, the piston (link 3) reciprocates inside the cylinders forming link 1.



Crank and slotted lever quick return motion mechanism.

This mechanism is mostly used in shaping machines, slotting machines and in rotary internal combustion engines.

In this mechanism, the link AC (i.e. link 3) forming the turning pair is fixed, as shown in Fig. The link 3 corresponds to the connecting rod of a reciprocating steam engine. The driving crank CB revolves with uniform angular speed about the fixed centre C. A sliding block attached to the crank pin at B slides along the slotted bar AP and thus causes AP to oscillate about the pivoted point A. A short link PR transmits the motion from AP to the ram which carries the tool

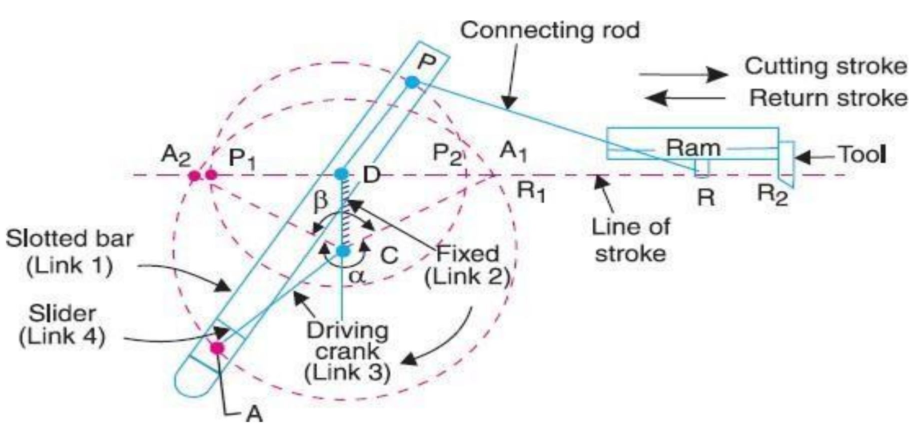


and reciprocates along the line of stroke R1R2. The line of stroke of the ram (i.e. R1R2) is perpendicular to AC produced.

In the extreme positions, AP1 and AP2 are tangential to the circle and the cutting tool is at the end of the stroke. The forward or cutting stroke occurs when the crank rotates from the position CB1 to CB2 (or through an angle) in the clockwise direction. The return stroke occurs when the crank rotates from the position CB2 to CB1 (or through angle) in the clockwise direction. Since the crank has uniform angular speed.

3. Whitworth quick return motion mechanism.

This mechanism is mostly used in shaping and slotting machines. In this mechanism, the link CD (link 2) forming the turning pair is fixed, as shown in Fig. The link 2 corresponds to a crank in a reciprocating steam engine.



The driving crank CA (link 3) rotates at a uniform angular speed. The slider (link 4) attached to the crank pin at A slides along the slotted bar PA (link 1) which oscillates at a pivoted point D. The connecting rod PR carries the ram at R to which a cutting tool is fixed. The motion of the tool is constrained along the line RD produced, i.e. along a line passing through D and perpendicular to CD

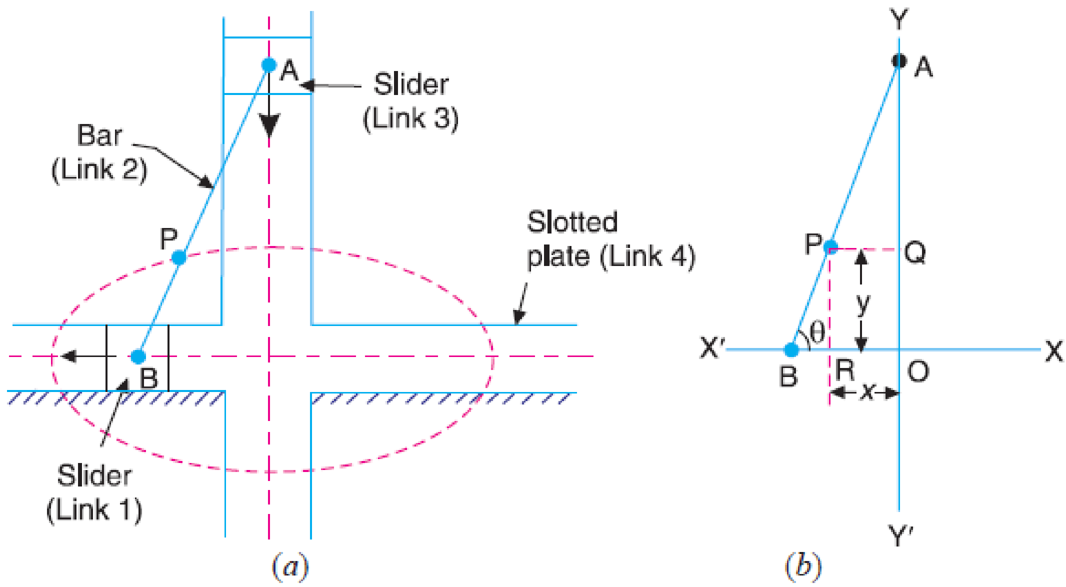
Double Slider Crank Chain

A kinematic chain which consists of two turning pairs and two sliding pairs is known as double slider crank chain

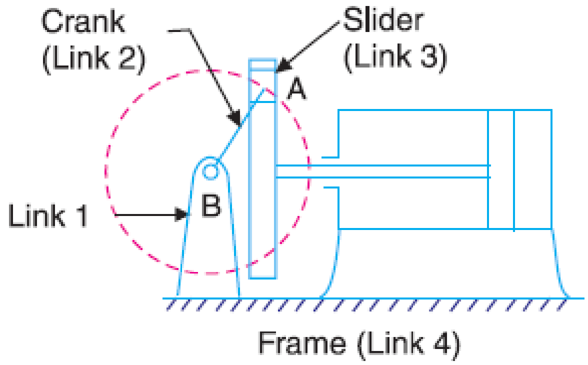
Inversions of Double Slider Crank Chain

The following three inversions of a double slider crank chain are important from the subject point of view:

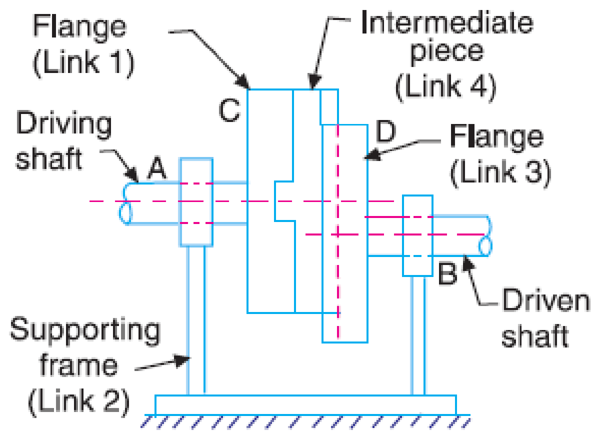
1. **Elliptical trammels.** It is an instrument used for drawing ellipses. This inversion is obtained by fixing the slotted plate (link 4), as shown in Fig. The fixed plate or link 4 has two straight grooves cut in it, at right angles to each other. The link 1 and link 3, are known as sliders and form sliding pairs with link 4. The link AB (link 2) is a bar which forms turning pair with links 1 and 3. When the links 1 and 3 slide along their respective grooves, any point on the link 2 such as P traces out an ellipse on the surface of link 4, as shown in Fig. (a). A little consideration will show that AP and BP are the semi-major axis and semi-minor axis of the ellipse respectively.



2. **Scotch yoke mechanism.** This mechanism is used for converting rotary motion into a reciprocating motion. The inversion is obtained by fixing either the link 1 or link 3. In Fig. link1 is fixed. In this mechanism, when the link 2 (which corresponds to crank) rotates about B as centre, the link 4 (which corresponds to a frame) reciprocates. The fixed link 1 guides the frame.



3. Oldham's coupling. An oldham's coupling is used for connecting two parallel shafts whose axes are at a small distance apart. The shafts are coupled in such a way that if one shaft rotates, the other shaft also rotates at the same speed. This inversion is obtained by fixing the link 2, as shown in Fig. (a). The shafts to be connected have two flanges (link 1 and link 3) rigidly fastened at their ends by forging.



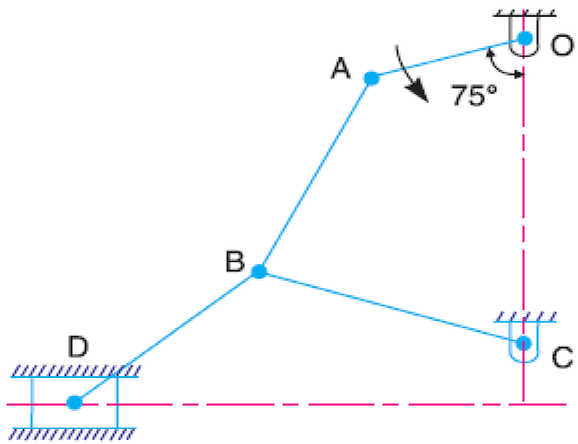
(*a*)

Experiment No: 2

One A3 size sheet of Velocity problems by relative velocity method.

(Minimum 4 problems)

Example:1 In Fig., the angular velocity of the crank OA is 600 r.p.m. Determine the linear velocity of the slider D and the angular velocity of the link BD, when the crank is inclined at an angle of 75° to the vertical. The dimensions of various links are : OA = 28 mm ; AB = 44 mm ; BC 49 mm ; and BD = 46 mm. The centre distance between the centres of rotation O and C is 65 mm. The path of travel of the slider is 11 mm below the fixed point C. The slider moves along a horizontal path and OC is vertical.



Solution. Given: N_{AO} = 600 r.p.m. or $\omega_{AO} = 2\,\pi \times 600/60 = 62.84$ rad/s

Since OA = 28 mm = 0.028 m, therefore velocity of A with respect to O or velocity of A (because O is a fixed point)

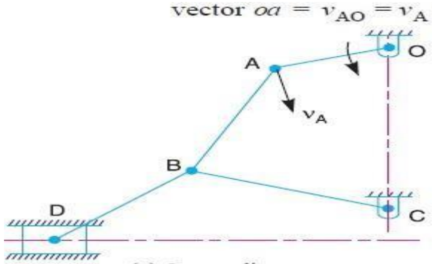
$$v_{AO} = v_A = \omega_{AO} \times OA = 62.84 \times 0.028 = 1.76 \text{ m/s}$$

Linear velocity of the slider D

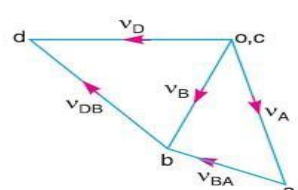
First of all draw the space diagram, to some suitable scale, as shown in Fig. (a)Now the velocity diagram, as shown in Fig. (b), is drawn as discussed below:

1. Since the points O and C are fixed, therefore these points are marked as one point, in the velocity diagram. Now from point o, draw vector oa perpendicular to OA, to some suitable scale, to represent the velocity of A with respect to O or simply velocity of A such that.

$$\text{vector } oa = v_{AO} = v_A = 1.76 \text{ m/s}$$



(a) Space diagram.



(b) Velocity diagram.

2. From point a, draw vector ab perpendicular to AB to represent the velocity of B with respect A (i.e. vBA) and from point c, draw vector cb perpendicular to CB to represent the velocity of B with respect to C or simply velocity of B (i.e. vBC or vB). The vectors ab and cb intersect at b.
3. From point b, draw vector bd perpendicular to BD to represent the velocity of D with respect to B (i.e. vDB) and from point o, draw vector od parallel to the path of motion of the slider D which is horizontal, to represent the velocity of D (i.e. vD). The vectors bd and od intersect at d. By measurement, we find that velocity of the slider D,

$$V_D = \text{vector } od = 1.6 \text{ m/s}$$

Angular velocity of the link BD

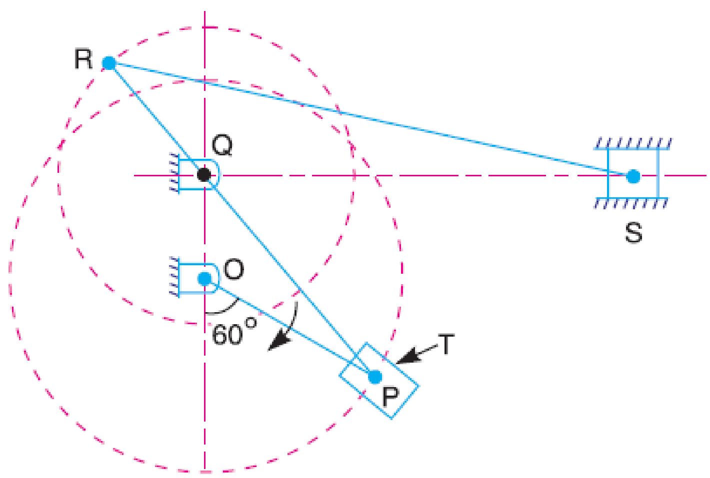
By measurement from velocity diagram, we find that velocity of D with respect to B,

$$v_{DB} = \text{vector } bd = 1.7 \text{ m/s}$$

Since the length of link $BD = 46$ mm $= 0.046$ m, therefore angular velocity of the link BD,

$$\omega_{BD} = \frac{v_{DB}}{BD} = \frac{1.7}{0.046} = 36.96 \text{ rad/s (Clockwise about } B\text{)} \quad \textbf{Ans.}$$

Example 2. Fig. shows the structure of Whitworth quick return mechanism used in reciprocating machine tools. The various dimensions of the tool are as follows : OQ = 100 mm ; OP = 200 mm, RQ =150 mm and RS = 500 mm. The crank OP makes an angle of 60° with the vertical. Determine the velocity of the slider S (cutting tool) when the crank rotates at 120 r.p.m. clockwise. Find also the angular velocity of the link RS and the velocity of the sliding block Ton the slotted lever QT.



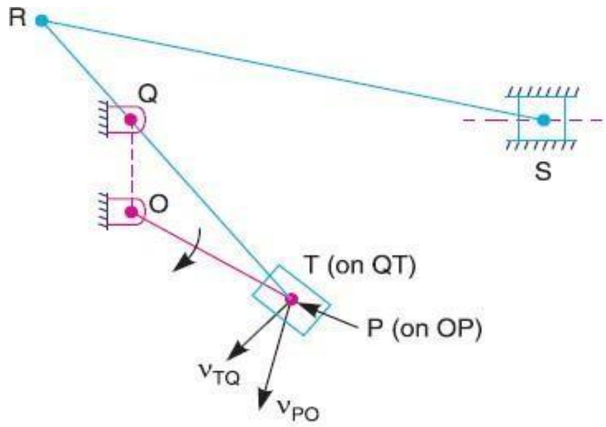
Solution. Given : N_{PO} = 120 r.p.m. or $\omega_{PO} = 2\pi \times 120/60 = 12.57$ rad/s

Since the crank OP = 200 mm = 0.2 m, therefore velocity of P with respect to O or velocity of P (because O is a fixed point)

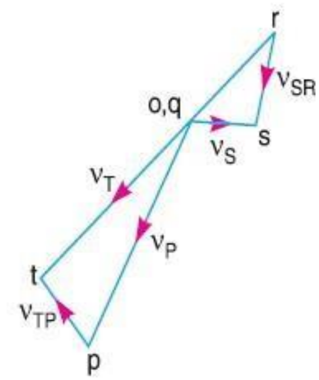
$$v_{PO} = v_P = \omega_{PO} \times OP = 12.57 \times 0.2 = 2.514 \text{ m/s}$$

Velocity of slider S (cutting tool)
First of all draw the space diagram, to some suitable scale, as shown in Fig. (a). Now the velocity diagram, as shown in Fig. (b) is drawn as discussed below:

1. Since O and Q are fixed points, therefore they are taken as one point in the velocity diagram. From point o, draw vector op perpendicular to OP, to some suitable scale, to represent the velocity of P with respect to O or simply velocity of P, such that.



(*a*) Space diagram.



(*b*) Velocity diagram.

2. From point q, draw vector qt perpendicular to QT to represent the velocity of T with respect to Q or simply velocity of T (i.e. vTQ or vT) and from point p draw vector pt parallel to the path of motion of T (which is parallel to TQ) to represent the velocity of T with respect to P (i.e. vTP). The vectors qt and pt intersect at t.

3. Since the point R lies on the link TQ produced, therefore divide the vector tq at r in the sameratio as R divides TQ, in the space diagram. In other words

$$qr/qt = QR/QT$$

The vector qr represents the velocity of R with respect to Q or velocity of R (i.e.vRQ or vR)

4. From point r, draw vector rs perpendicular to RS to represent the velocity of S with respect to R and from point o draw vector or parallel to the path of motion of S (which is parallel to QS) to represent the velocity of S (i.e vS). The vectors rs and os intersect at s. By measurement, we find that velocity of the slider S (cutting tool)

$$v_S = \text{vector os} = 0.8 \text{ m/s}$$

Angular velocity of link RS From the velocity diagram, we find that the linear velocity of the link RS $\quad v_{SR} = \text{vector rs} = 0.96 \text{ m/s}$

Since the length of link RS = 500 mm = 0.5 m, therefore angular velocity of link RS

$$v_{SR} = \text{vector } rs = 0.96 \text{ m/s}$$

Since the length of link *RS* = 500 mm = 0.5 m, therefore angular velocity of link *RS*,

$$\omega_{RS} = \frac{v_{SR}}{RS} = \frac{0.96}{0.5} = 0.92 \text{ rad/s (Clockwise about } R\text{)} \textbf{ Ans.}$$

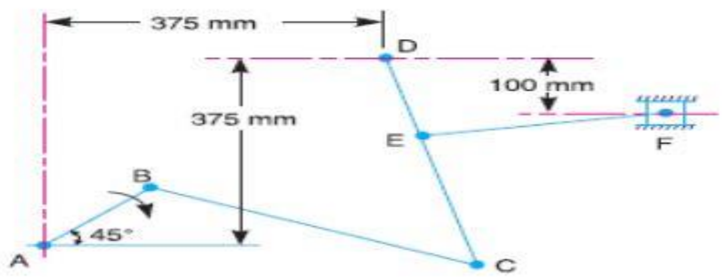
Velocity of the sliding block T on the slotted lever QT

Since the block *T* moves on the slotted lever with respect to *P*, therefore velocity of the sliding block *T* on the slotted lever *QT*,

$$v_{TP} = \text{vector } pt = 0.85 \text{ m/s} \quad \textbf{Ans.} \quad \text{. . . (By measurement)}$$

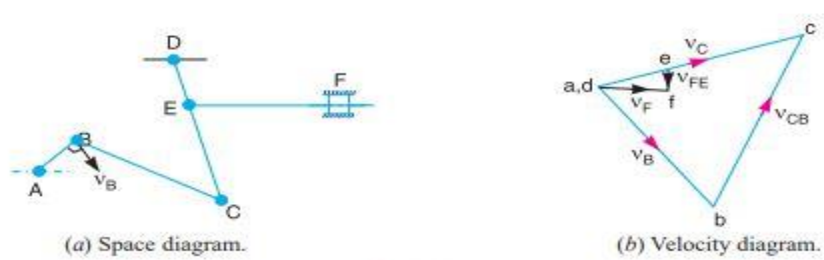
Example 3. The mechanism, as shown in Fig. 7.11, has the dimensions of various links as follows : AB = DE = 150 mm ; BC = CD = 450 mm ; EF = 375 mm

The crank AB makes an angle of 45° with the horizontal and rotates about A in the clockwise direction at a uniform speed of 120 r.p.m. The lever DC oscillates about the fixed point D, which is connected to AB by the coupler BC. The block F moves in the horizontal guides, being driven by the link EF. Determine: 1. velocity of the block F, 2. angular velocity of DC, and 3. rubbing speed at the pin C which is 50 mm in diameter.



Solution. Given : NBA = 120 r.p.m. or ωBA = 2 π × 120/60 = 4 π rad/s Since the crank length A B = 150 mm = 0.15 m, therefore velocity of B with respect to A or simply velocity of B (because A is a fixed point),

vBA = vB = ωBA × AB = 4 π × 0.15 = 1.885 m/s . . . (Perpendicular to A B)



(*a*) Space diagram. (*b*) Velocity diagram.

1. Velocity of the block F First of all draw the space diagram, to some suitable scale, as shown in Fig. 7.12 (a). Now the velocity diagram, as shown in Fig. 7.12 (b), is drawn as discussed below:

1. Since the points A and D are fixed, therefore these points are marked as one point* as shown in Fig. 7.12 (b). Now from point a, draw vector ab perpendicular to A B, to some suitable scale, to represent the velocity of B with respect to A or simply velocity of B, such that vector ab = vBA = vB = 1.885 m/s

2. The point C moves relative to B and D, therefore draw vector bc perpendicular to BC to represent the velocity of C with respect to B (i.e. vCB), and from point d, draw vector dc perpendicular to DC to represent the velocity of C with respect to D or simply velocity of C (i.e. vCD or vC). The vectors bc and dc intersect at c.

3. Since the point E lies on DC, therefore divide vector dc in e in the same ratio as E divides CD in Fig. 7.12 (a). In other words ce/cd = CE/CD The point e on dc may be marked in the same manner as discussed in Example 7.2.

4. From point e, draw vector ef perpendicular to EF to represent the velocity of F with respect to E (i.e. vFE) and from point d draw vector df parallel to the path of motion of F, which is horizontal, to represent the velocity of F i.e. vF. The vectors ef and df intersect at f. By measurement, we find that velocity of the block F, vF = vector df = 0.7 m/s Ans

2. Angular velocity of DC

By measurement from velocity diagram, we find that velocity of C with respect to D,

vCD = vector dc = 2.25 m/s

Since the length of link DC = 450 mm = 0.45 m, therefore angular velocity of DC

$$\omega_{DC} = \frac{v_{CD}}{DC} = \frac{2.25}{0.45} = 5 \text{ rad/s}$$

3. Rubbing speed at the pin C

We know that diameter of pin at C,

dC = 50 mm = 0.05 m or Radius , rC = 0.025 m

From velocity diagram, we find that velocity of C with respect to B,

vCB = vector bc = 2.25 m/s . . . (By measurement)

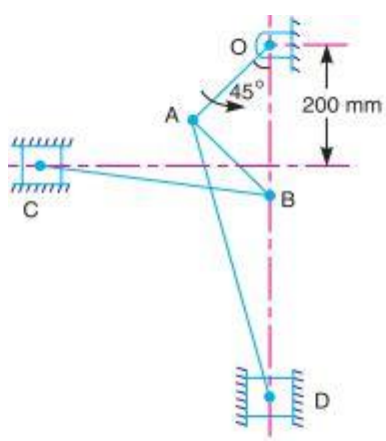
Length BC = 450 mm = 0.45 m

∴ Angular velocity of BC,

$$\omega_{CB} = \frac{v_{CB}}{BC} = \frac{2.25}{0.45} = 5 \text{ rad/s}$$

We know that rubbing speed at the pin C = (ωCB – ωCD) r C = (5 – 5) 0.025 = 0

Example 4. The dimensions of the various links of a pneumatic riveter, as shown in Fig. 7.26, are as follows : OA = 175 mm ; AB = 180 mm ; AD = 500 mm ; and BC = 325 mm. Find the velocity ratio between C and ram D when OB is vertical. What will be the efficiency of the machine if a load of 2.5 kN on the piston C causes a thrust of 4 kN at the ram D ?



Solution. Given : WC = 2.5 kN = 2500 N ; WD = 4 kN = 4000 N Let

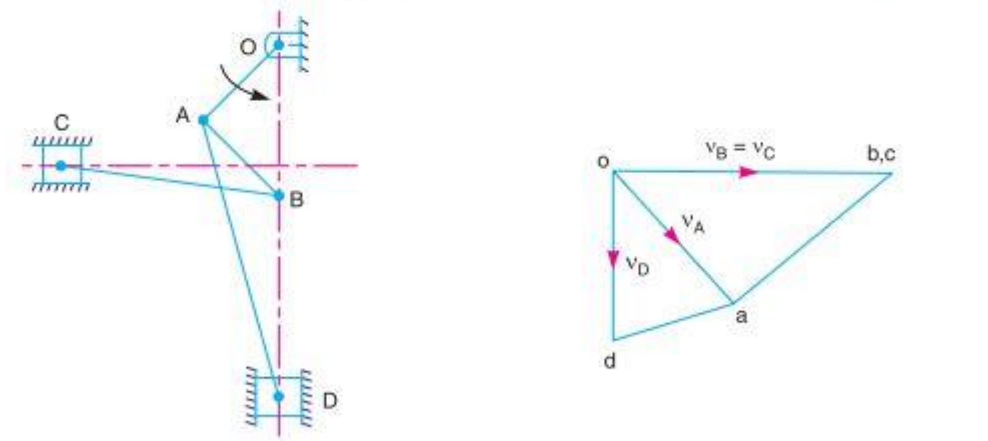
N = Speed of crank OA.

∴Angular velocity of crank OA,

ωAO = 2 π N/60 rad/s

Since the length of crank OA = 175 mm = 0.175 m, therefore velocity of A with respect to O (or velocity of A) (because O is a fixed point),

$$v_{AO} = v_A = \frac{2\pi N}{60} \times 0.175 = 0.0183\ N \text{ m/s} \qquad \text{. . . (Perpendicular to } OA\text{)}$$



(*a*) Space diagram. (*b*) Velocity diagram.

Velocity ratio between C and the ram D

First of all draw the space diagram, to some suitable scale, as shown in Fig. 7.27 (a), Now the velocity diagram, as shown in Fig. 7.27 (b), is drawn as discussed below :

1. Draw vector oa perpendicular to OA to represent the velocity of A (i.e. vA) such that vector oa = vA = 0.0183 N m/s Since the speed of crank (N) is not given, therefore let we take vector oa = 20 mm.

2. From point a, draw a vector ab perpendicular to A B to represent the velocity of B with respect to A (i.e. vBA), and from point o draw vector ob perpendicular to OB to represent the velocity of B with respect to A or simply velocity of B (i.e. vBO or vB). The vectors ab and ob intersect at b.

3. Now from point b, draw vector bc perpendicular to BC to represent the velocity of C with respect to B (i.e. vCB) and from point o draw vector oc parallel to the path of motion of C to represent the velocity of C (i.e. vC). The vectors bc and oc intersect at c. We see from Fig. 7.27 (b) that the points b and c coincide. Therefore velocity of B with respect to C is zero and velocity of B is equal to velocity of C, i.e.

vBC = 0 . . . (b and c coincide) and vB = vC . . . (vector ob = vector oc)

4. From point a, draw vector ad perpendicular to AD to represent velocity of D with respect to A i.e. vDA, and from point o draw vector ob parallel to the path of motion of D to represent the velocity of D i.e. vD. The vectors ad and od intersect at d.

By measurement from velocity diagram, we find that velocity of C,

vC = vector oc = 35 mm and velocity of D, vD = vector od = 21 mm

∴Velocity ratio between C and the ram D = vC /vD = 35/21 = 1.66

Efficiency of the machine

Let η = Efficiency of the machine,

We know that work done on the piston C or input,

$$= W_C \times v_C = 2500\, v_C$$

and work done by the ram D or output,

$$= W_D \times v_D = 4000\, v_D$$

$$\therefore \quad \eta = \frac{\text{Output}}{\text{Input}} = \frac{4000\, v_D}{2500\, v_C} = \frac{4000}{2500} \times \frac{1}{1.66} \qquad \ldots \left(\because \frac{v_C}{v_D} = 1.66 \right)$$

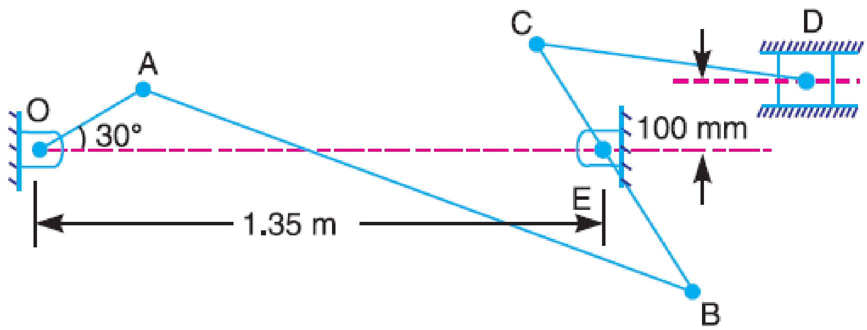
$$= 0.96 \text{ or } 96\% \textbf{ Ans.}$$

Experiment No: 3

One A3 size sheet of Velocity problems by Klien's construction and Instantaneous center method.

(Minimum 4 problems)

Example 1. A mechanism, as shown in Fig. has the following dimensions:
OA = 200 mm; AB = 1.5 m; BC = 600 mm; CD = 500 mm and BE = 400 mm. Locate all the instantaneous centres.
If crank OA rotates uniformly at 120 r.p.m. clockwise, find 1. the velocity of B, C and D,
2. the angular velocity of the links AB, BC and CD.



Solution:

Given : $N_{OA} = 120$ r.p.m. or $\omega_{OA} = 2\,\pi \times 120/60 = 12.57$ rad/s
Since the length of crank OA = 200 mm = 0.2 m, therefore linear velocity of crank OA,
$V_{OA} = V_A = \omega_{OA} \times OA = 12.57 \times 0.2 = 2.514$ m/s
Location of instantaneous centres
The instantaneous centres are located as discussed below:
1. Since the mechanism consists of six links (i.e. n = 6), therefore the number of instantaneous centres,

$N = n(n-1)/2 = 15$ as $n=6$

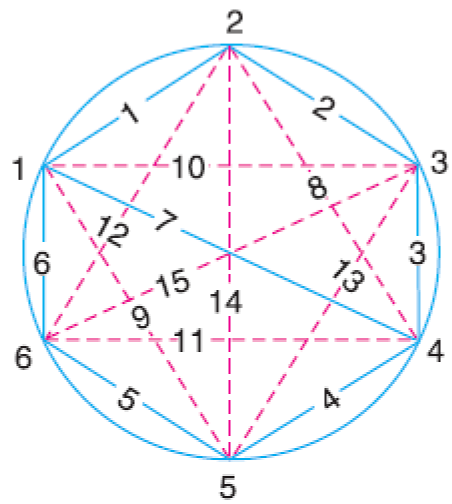
link	1	2	3	4	5	6
I.C.	12	23	34	45	56	
	13	24	35	46		
	14	25	36			
	15	26				
	16					

2. Make a list of all the instantaneous centres in a mechanism. Since the mechanism has 15 instantaneous centres, therefore these centres are listed in the following book keeping table.

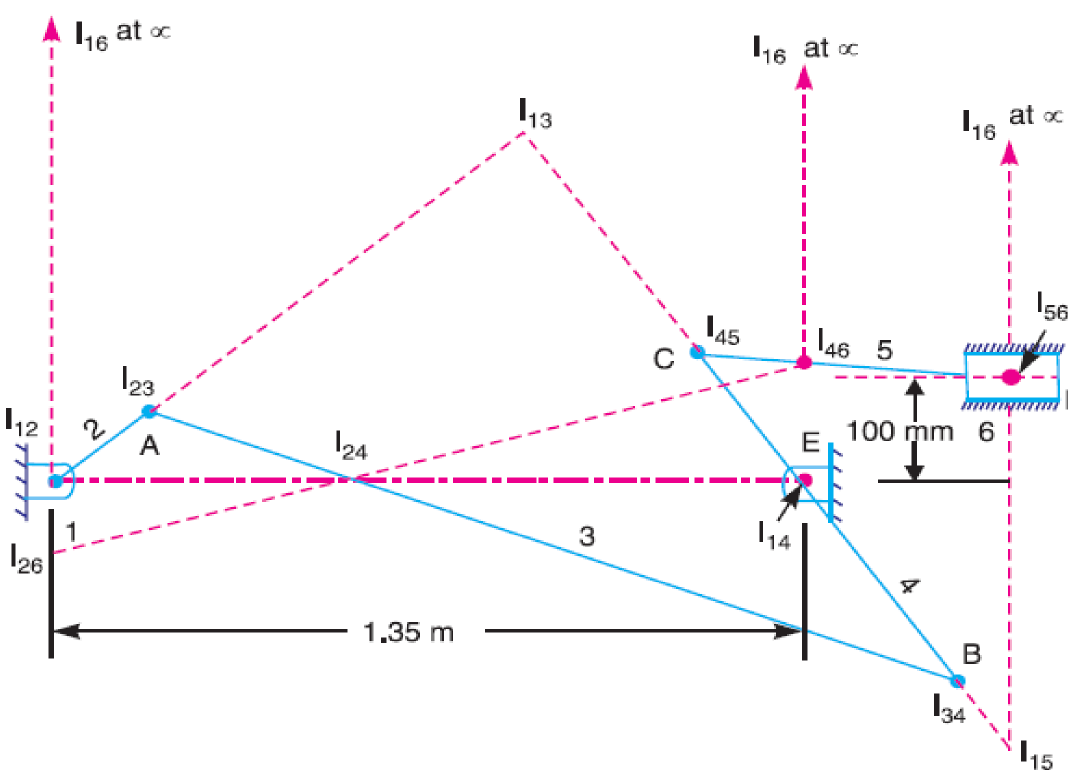
3. Locate the fixed and permanent instantaneous centres by inspection. These centres are I12 I23, I34, I45, I56, I16 and I14 as shown in Fig. 6.16.

4. Locate the remaining neither fixed nor permanent instantaneous centres by Aronhold Kennedy's theorem. Draw a circle and mark points equal to the number of links such as 1, 2, 3, 4, 5 and 6 as shown in Fig. 6.17. Join the points 12, 23, 34, 45, 56, 61 and 14 to indicate the centres I12, I23, I34, I45, I56, I16 and I14 respectively.

5. Join point 2 to 4 by a dotted line to form the triangles 1 2 4 and 2 3 4. The side 2 4, common to both triangles, is responsible for completing the two triangles. Therefore the instantaneous centre I24 lies on the intersection of I12 I14 and I23 I34 produced if necessary. Thus centre I24 is located. Mark number 8 on the dotted line 24 (because seven centres have already been located).



6. Now join point 1 to 5 by a dotted line to form the triangles 1 4 5 and 1 5 6. The side 1 5, common to both triangles, is responsible for completing the two triangles. Therefore the instantaneous centre I15 lies on the intersection of I14 I45 and I56 I16 produced if necessary. Thus centre I15 is located. Mark number 9 on the dotted line 1 5.



7. Join point 1 to 3 by a dotted line to form the triangles 1 2 3 and 1 3 4. The side 1 3, common to both triangles, is responsible for completing the two triangles. Therefore the instantaneous centre I13 lies on the intersection I12 I23 and I34 I14 produced if necessary. Thus centre I13 is located. Mark number 10 on the dotted line 1 3.

8. Join point 4 to 6 by a dotted line to form the triangles 4 5 6 and 1 4 6. The side 4 6, common to both triangles, is responsible for completing the two triangles. Therefore, centre I46 lies on the intersection of I45 I56 and I14 I16. Thus centre I46 is located. Mark number 11 on the dotted line 4 6.

9. Join point 2 to 6 by a dotted line to form the triangles 1 2 6 and 2 4 6. The side 2 6, common to both triangles, is responsible for completing the two triangles. Therefore, centre I26 lies on the intersection of lines joining the points I12 I16 and I24 I46. Thus centre I26 is located. Mark number 12 on the dotted line 2 6.

10. In the similar way the thirteenth, fourteenth and fifteenth instantaneous centre (i.e. I35, I25 and I36) may be located by joining the point 3 to 5, 2 to 5 and 3 to 6 respectively.
By measurement, we find that
I13 A = 840 mm = 0.84 m ; I13 B = 1070 mm = 1.07 m ; I14 B = 400 mm = 0.4 m ;
I14 C = 200 mm = 0.2 m ; I15 C = 740 mm = 0.74 m ; I15 D = 500 mm = 0.5 m
1. Velocity of points B, C and D
Let V_B, V_C and V_D = Velocity of the points B, C and D respectively.

Example: 2

Fig. shows a sewing needle bar mechanism O1ABO2CD wherein the different dimensions are as follows: Crank O1A = 16 mm; β = 45°; Vertical distance between O1 and O2 = 40 mm; Horizontal distance between O1 and O2 = 13 mm; O2 B = 23 mm; AB = 35 mm; Angle O2 BC = 90°; BC = 16 mm; CD = 40 mm. D lies vertically below O1.Find the velocity of needle at D for the given configuration. The crank O1A rotates at 400 r.p.m.

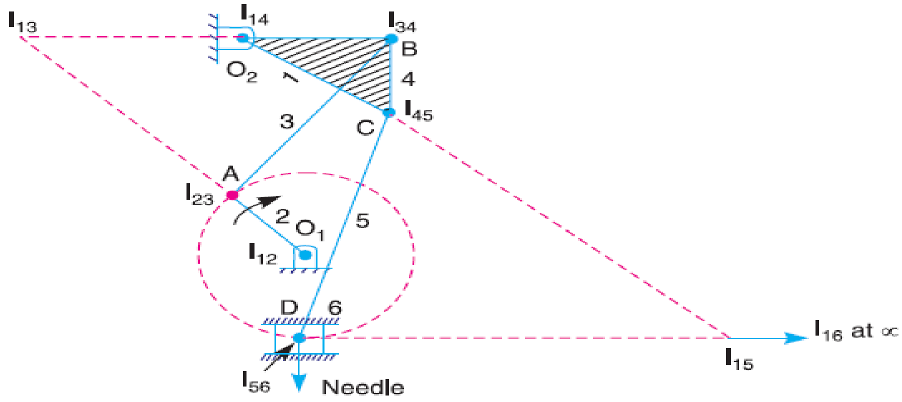
Solution. Given : NO1A = 400 r.p.m or ωO1A = 2□□× 400/60 = 41.9 rad/s ; O1 A = 16 mm = 0.016 m

We know that linear velocity of the crank O1A, vO1A = vA = □O1A × O1A = 41.9 × 0.016 = 0.67 m/s Now let us locate the required instantaneous centrs as discussed below :
1. Since the mechanism consists of six links (i.e. n = 6), therefore number of instantaneous centers

$$N = \frac{n(n-1)}{2} = \frac{6(6-1)}{2} = 15$$

2. Since the mechanism has 15 instantaneous centres, therefore these centres may be listed in the book keeping table, as discussed in Example
3. Locate the fixed and permanent instantaneous centres by inspections. These centres are I12, I23, I34, I45, I56, I16 and I14, as shown in fig.
4. Locate the remaining neither fixed nor permanent instantaneous centres by Aronhold Kennedy's theorem. This is done by circle diagram as shown in Fig. Mark six points on the

circle (i.e. equal to the number of links in a mechanism) and join 1 to 2, 2 to 3, 3 to 4, 4 to 5, 5 to 6, 6 to 1 and 1 to 4 to indicate the fixed and permanent instantaneous centres i.e. I12, I23, I34, I45, I56,I16 and I14 respectively.



5. Join 1 to 3 by a dotted line to form two triangles 1 2 3 and 1 3 4. The side 1 3, common to both the triangles, is responsible for completing the two triangles. Therefore the instantaneous centre I13 lies on the intersection of I12 I23 and I14 I34 produced if necessary. Thus centre I13 is located. Mark number 8 (because seven centres have already been located) on the dotted line 1 3.
6. Join 1 to 5 by a dotted line to form two triangles 1 5 6 and 1 4 5. The side 1 5, common to both the triangles, is responsible for completing the two triangles. Therefore the instantaneous centre I15 lies on the intersection of I16 I56 and I14 I45 produced if necessary. Thus centre I15 is located. Mark number 9 on the dotted line 1 5.
Note: For the given example, we do not require other instantaneous centres.

By measurement, we find that

I13A = 41 mm = 0.041 m ; I13 B = 50 mm = 0.05 m ; I14 B = 23 mm = 0.023 m ;
I14 C= 28 mm = 0.028 m ; I15 C = 65 mm = 0.065 m ; I15 D = 62 mm = 0.062 m

v_B = Velocity of point B,
v_C = Velocity of point C, and
v_D = Velocity of the needle at D.

We know that $\dfrac{v_A}{I_{13}A} = \dfrac{v_B}{I_{13}B}$ ...(Considering centre I_{13})

$$\therefore \quad v_B = \frac{v_A}{I_{13}A} \times I_{13}B = \frac{0.67}{0.041} \times 0.05 = 0.817 \text{ m/s}$$

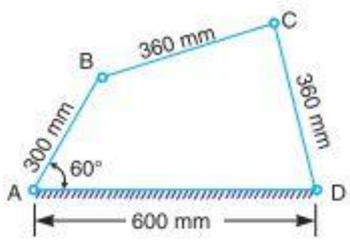
and $\dfrac{v_B}{I_{14}B} = \dfrac{v_C}{I_{14}C}$ ...(Considering centre I_{14})

$$\therefore \quad v_C = \frac{v_B}{I_{14}B} \times I_{14}C = \frac{0.817}{0.023} \times 0.028 = 0.995 \text{ m/s}$$

Similarly, $\dfrac{v_C}{I_{15}C} = \dfrac{v_D}{I_{15}D}$ ...(Considering centre I_{15})

$$\therefore \quad v_D = \frac{v_C}{I_{15}C} \times I_{15}D = \frac{0.995}{0.065} \times 0.062 = 0.95 \text{ m/s} \quad \textbf{Ans.}$$

Example:3 In a pin jointed four bar mechanism, as shown in Fig. 6.9, AB = 300 mm, BC = CD = 360 mm, and AD = 600 mm. The angle BAD = 60°. The crank AB rotates uniformly at 100 r.p.m. Locate all the instantaneous centres and find the angular velocity of the link BC.



Solution. Given : NAB = 100 r.p.m or

$\omega AB = 2\pi \times 100/60 = 10.47$ rad/s

Since the length of crank A B = 300 mm = 0.3 m,

therefore velocity of point B on link A B

$vB = \omega AB \times AB = 10.47 \times 0.3 = 3.141$ m/s

Location of instantaneous centres

The instantaneous centres are located as discussed below:

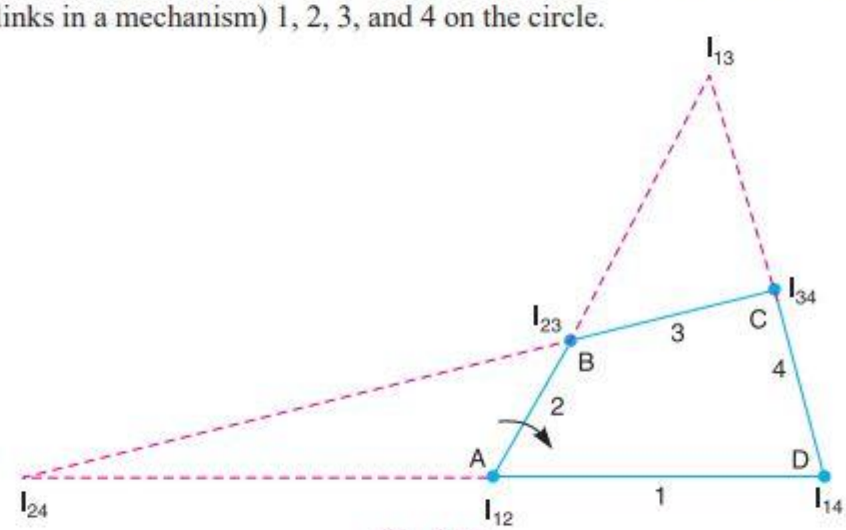
1. Since the mechanism consists of four links (*i.e.* $n = 4$), therefore number of instantaneous centres,

$$N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$

2. For a four bar mechanism, the book keeping table may be drawn as discussed in Art. 6.10.

3. Locate the fixed and permanent instantaneous centres by inspection. These centres are I_{12}, I_{23}, I_{34} and I_{14}, as shown in Fig. 6.10.

4. Locate the remaining neither fixed nor permanent instantaneous centres by Aronhold Kennedy's theorem. This is done by circle diagram as shown in Fig. 6.11. Mark four points (equal to the number of links in a mechanism) 1, 2, 3, and 4 on the circle.



5. Join points 1 to 2, 2 to 3, 3 to 4 and 4 to 1 to indicate the instantaneous centres already located *i.e.* I_{12}, I_{23}, I_{34} and I_{14}.

6. Join 1 to 3 to form two triangles 1 2 3 and 3 4 1. The side 13, common to both triangles, is responsible for completing the two triangles. Therefore the instantaneous centre I_{13} lies on the intersection of the lines joining the points I_{12} I_{23} and I_{34} I_{14} as shown in Fig. 6.10. Thus centre I_{13} is located. Mark number 5 (because four instantaneous centres have already been located) on the dotted line 1 3.

7. Now join 2 to 4 to complete two triangles 2 3 4 and 1 2 4. The side 2 4, common to both triangles, is responsible for completing the two triangles. Therefore centre I_{24} lies on the intersection of the lines joining the points I_{23} I_{34} and I_{12} I_{14} as shown in Fig. 6.10. Thus centre I_{24} is located. Mark number 6 on the dotted line 2 4. Thus all the six instantaneous centres are located.

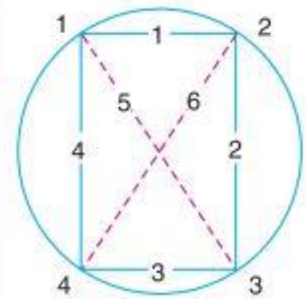


Fig. 6.11

Angular velocity of the link BC

Let ωBC = Angular velocity of the link BC.

Since B is also a point on link BC, therefore velocity of point B on link BC,

vB = ωBC × I 13 B

By measurement, we find that I13 B = 500 mm = 0.5 m

$$\omega_{BC} = \frac{v_B}{I_{13}B} = \frac{3.141}{0.5} = 6.282 \text{ rad/s} \quad \textbf{Ans.}$$

Example:4 Locate all the instantaneous centres of the slider crank mechanism as shown in Fig. 6.12. The lengths of crank OB and connecting rod AB are 100 mm and 400 mm respectively. If the crank rotates clockwise with an angular velocity of 10 rad/s, find: 1. Velocity of the slider A, and 2. Angular velocity of the connecting rod AB.

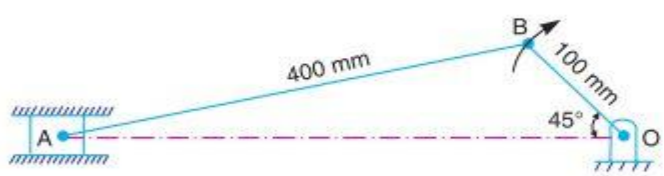


Fig. 6.12

Solution. Given : $\omega_{OB} = 10$ rad/ s; $OB = 100$ mm $= 0.1$ m

We know that linear velocity of the crank OB,

$$v_{OB} = v_B = \omega_{OB} \times OB = 10 \times 0.1 = 1 \text{ m/s}$$

Location of instantaneous centres

The instantaneous centres in a slider crank mechanism are located as discussed below:

1. Since there are four links (*i.e.* $n = 4$), therefore the number of instantaneous centres,

$$N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$

2. For a four link mechanism, the book keeping table may be drawn as discussed in Art. 6.10.

3. Locate the fixed and permanent instantaneous centres by inspection. These centres are I_{12}, I_{23} and I_{34} as shown in Fig. 6.13. Since the slider (link 4) moves on a straight surface (link 1), therefore the instantaneous centre I_{14} will be at infinity.

4. Locate the other two remaining neither fixed nor permanent instantaneous centres, by Aronhold Kennedy's theorem. This is done by circle diagram as shown in Fig. 6.14. Mark four points 1, 2, 3 and 4 (equal to the number of links in a mechanism) on the circle to indicate I_{12}, I_{23}, I_{34} and I_{14}.

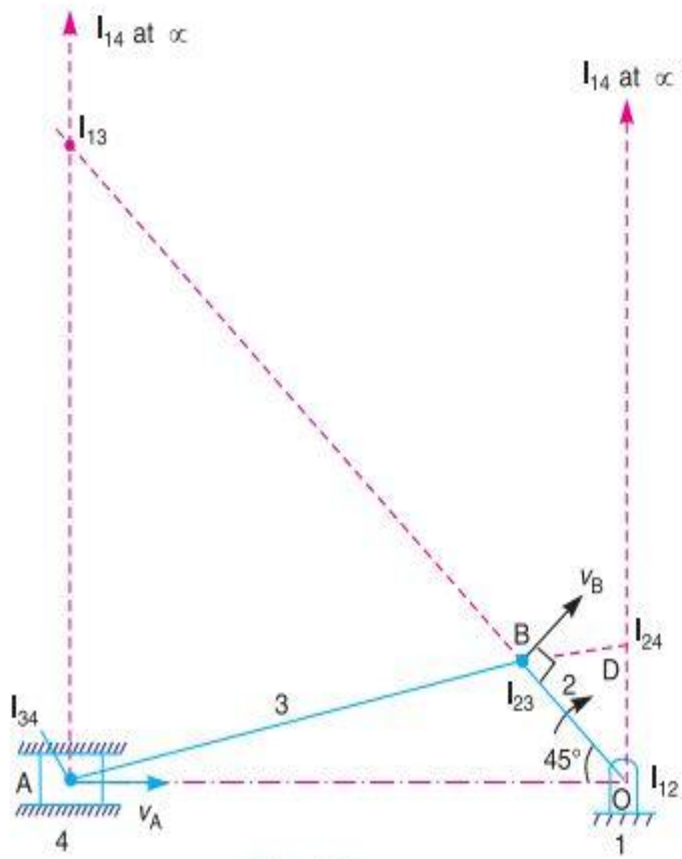


Fig. 6.13

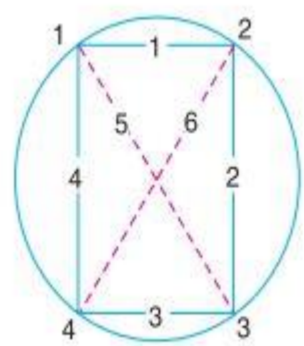


Fig. 6.14

5. Join 1 to 3 to form two triangles 1 2 3 and 3 4 1 in the circle diagram. The side 1 3, common to both triangles, is responsible for completing the two triangles. Therefore the centre I_{13} will lie on the intersection of $I_{12}\,I_{23}$ and $I_{14}\,I_{34}$, produced if necessary. Thus centre I_{13} is located. Join 1 to 3 by a dotted line and mark number 5 on it.

6. Join 2 to 4 by a dotted line to form two triangles 2 3 4 and 1 2 4. The side 2 4, common to both triangles, is responsible for completing the two triangles. Therefore the centre I_{24} lies on the intersection of $I_{23}\,I_{34}$ and $I_{12}\,I_{14}$. Join 2 to 4 by a dotted line on the circle diagram and mark number 6 on it. Thus all the six instantaneous centres are located.

By measurement, we find that

$$I_{13}\,A = 460 \text{ mm} = 0.46 \text{ m} \;;\text{ and } I_{13}\,B = 560 \text{ mm} = 0.56 \text{ m}$$

1. ***Velocity of the slider A***

Let v_A = Velocity of the slider A.

We know that $\dfrac{v_A}{I_{13}\,A} = \dfrac{v_B}{I_{13}\,B}$

or $v_A = v_B \times \dfrac{I_{13}\,A}{I_{13}\,B} = 1 \times \dfrac{0.46}{0.56} = 0.82$ m/s **Ans.**

2. ***Angular velocity of the connecting rod AB***

Let ω_{AB} = Angular velocity of the connecting rod $A\,B$.

We know that $\dfrac{v_A}{I_{13}\,A} = \dfrac{v_B}{I_{13}\,B} = \omega_{AB}$

$\therefore$ $\omega_{AB} = \dfrac{v_B}{I_{13}\,B} = \dfrac{1}{0.56} = 1.78$ rad/s **Ans.**

Note: The velocity of the slider A and angular velocity of the connecting rod $A\,B$ may also be determined as follows :

From similar triangles $I_{13}\,I_{23}\,I_{34}$ and $I_{12}\,I_{23}\,I_{24}$,

$$\frac{I_{12}\,I_{23}}{I_{13}\,I_{23}} = \frac{I_{23}\,I_{24}}{I_{23}\,I_{34}} \qquad ...(i)$$

and

$$\frac{I_{13}\,I_{34}}{I_{34}\,I_{23}} = \frac{I_{12}\,I_{24}}{I_{23}\,I_{24}} \qquad ...(ii)$$

We know that $\omega_{AB} = \dfrac{v_B}{I_{13}\,B} = \dfrac{\omega_{OB} \times OB}{I_{13}\,B}$ ...($\because\ v_B = \omega_{OB} \times OB$)

$$= \omega_{OB} \times \frac{I_{12}\,I_{23}}{I_{13}\,I_{23}} = \omega_{OB} \times \frac{I_{23}\,I_{24}}{I_{23}\,I_{34}} \qquad ...[\text{From equation } (i)] \;...(iii)$$

Also $v_A = \omega_{AB} \times I_{13}\,A = \omega_{OB} \times \dfrac{I_{23}\,I_{24}}{I_{23}\,I_{34}} \times I_{13}\,I_{34}$. ...[From equation (*iii*)]

$$= \omega_{OB} \times I_{12}\,I_{24} = \omega_{OB} \times OD \qquad ...[\text{From equation } (ii)]$$

Experiment No: 4

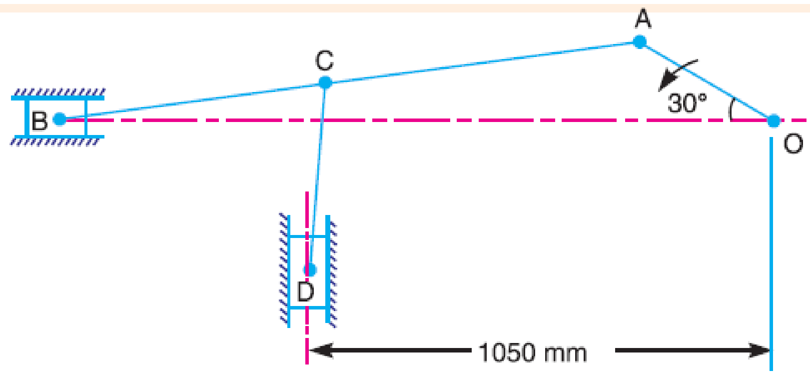
Solution of minimum two problems each on relative acceleration Method, involving Coriolis Acceleration and one each on short cut Methods

Example 1. In the mechanism, as shown in , the crank OA rotates at 20 r.p.m. anticlockwise and gives motion to the sliding blocks B and D. The dimensions of the various links are OA = 300 mm; AB = 1200 mm; BC = 450 mm and CD = 450 mm. For the given configuration, determine : 1. velocities of sliding at B and D, 2. Angular velocity of CD, 3. linear acceleration of D, and 4. angular acceleration of CD.

Solution:

Given : N_{AO} = 20 r.p.m. or $\omega_{AO} = 2\pi \times 20/60 = 2.1$ rad/s ; OA = 300 mm = 0.3 m ;
AB = 1200 mm = 1.2 m ; BC = CD = 450 mm = 0.45 m

We know that linear velocity of A with respect to O or velocity of A,
$V_{AO} = VA = \omega_{AO} \times OA = 2.1 \times 0.3 = 0.63$ m/s ...(Perpendicular to OA)



1. Velocities of sliding at B and D

First of all, draw the space diagram, to some suitable scale. Now the velocity diagram is drawn as discussed below:

1. Draw vector oa perpendicular to OA, to some suitable scale, to represent the velocity of A with respect to O (or simply velocity of A), such that
vector oa = $v_{AO} = v_A = 0.63$ m/s

2. From point a, draw vector ab perpendicular to AB to represent the velocity of B with respect to A (i.e. v_{BA}) and from point o draw vector ob parallel to path of motion B (which is along BO) to represent the velocity of B with respect to O (or simply velocity of B). The vectors ab and ob intersect at b.

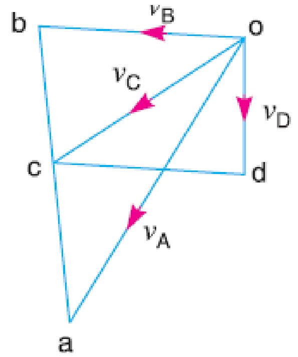
3. Divide vector ab at c in the same ratio as C divides AB in the space diagram. In other words, BC/CA = bc/ca

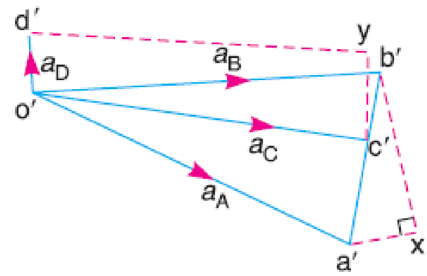
4. Now from point c, draw vector cd perpendicular to CD to represent the velocity of D with respect to C (i.e. v_{DC}) and from point o draw vector od parallel to the path of motion of D (which along the vertical direction) to represent the velocity of D.

By measurement, we find that velocity of sliding at B,
V_B = vector ob = 0.4 m/s .

and velocity of sliding at D, V_D = vector od = 0.24 m/s .
2. Angular velocity of CD
By measurement from velocity diagram, we find that velocity of D with respect to C,
V_{DC} = vector cd = 0.37 m/s
∴ Angular velocity of CD,

$\omega_{CD} = V_{DC}/CD = 0.37/0.45 = 0.82$ rad/s





3. Linear acceleration of D
We know that the radial component of the acceleration of A with respect to O or acceleration of A,

$a^r_{AO} = a_A = 1.231$ m/s^2

Radial component of the acceleration of B with respect to A,

$a^r_{BA} = 0.243$ m/s^2

Radial component of the acceleration of D with respect to C,

$a^r_{dc} = 0.304$ m/s^2

Now the acceleration diagram, as shown in Fig. 8.13 (c), is drawn as discussed below:
1. Draw vector o' a' parallel to OA, to some suitable scale, to represent the radial component of the acceleration of A with respect to O or simply the acceleration of A, such that

vector $o'a' = a^r_{OA} = a_A = 1.323$ m/s^2

2. From point a', draw vector a' x parallel to AB to represent the radial component of the acceleration of B with respect to A, such that

vector $a'x = a^r_{BA} = 0.243$ m/s

3. From point x, draw vector xb' perpendicular to AB to represent the tangential component of the acceleration of B with respect to A (i.e. BA a^t_{BA}) whose magnitude is not yet known.
4. From point o', draw vector o' b' parallel to the path of motion of B (which is along BO) to represent the acceleration of B (a_B). The vectors xb' and o' b' intersect at b'. Join a' b'. The vector a' b' represents the acceleration of B with respect to A.
5. Divide vector a' b' at c' in the same ratio as C divides AB in the space diagram. In other words, BC / B A = b' c'/b' a'
6. From point c', draw vector c'y parallel to CD to represent the radial component of the acceleration of D with respect to C, such that

vector $c'y = a^r_{DC} = 0.304$ m/s^2

7. From point y, draw yd' perpendicular to CD to represent the tangential component of

acceleration of D with respect to C (i.e. $a^t_{D\,C}$)whose magnitude is not yet known.
8. From point o', draw vector o' d' parallel to the path of motion of D (which is along the vertical direction) to represent the acceleration of D (a_D). The vectors yd' and o' d' intersect at d'.
By measurement, we find that linear acceleration of D,

$$a_D = \text{vector } o'\,d' = 0.16 \text{ m/s}^2 \text{ Ans.}$$

4. Angular acceleration of CD
From the acceleration diagram, we find that the tangential component of the acceleration of D with respect to C,

a^t_{DC}= yd′ =1.28 m/s^2 ...(By measurement)

∴ Angular acceleration of CD,

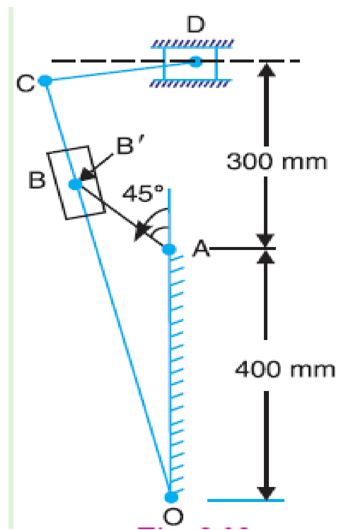
$$\alpha_{CD} = 2.84 \text{ rad/s}^2 \text{ (Clockwise)}$$

Example 2. A mechanism of a crank and slotted lever quick return motion is shown in Fig. If the crank rotates counter clockwise at 120 r.p.m., determine for the configuration shown, the velocity and acceleration of the ram D. Also determine the angular acceleration of the slotted lever. Crank, AB = 150 mm ; Slotted arm, OC = 700 mm and link CD = 200 mm.

Solution. Given : N_{BA} = 120 r.p.m or $\omega_{BA} = 2\pi \times 120/60 = 12.57$ rad/s ; AB = 150 mm = 0.15 m; OC = 700 mm = 0.7 m; CD = 200 mm = 0.2 m

We know that velocity of B with respect to A,

$V_{BA} = \omega_{BA} * AB = 1.9$ m/s ...(Perpendicular to AB)



Velocity of the ram D

First of all draw the space diagram, to some suitable scale, as shown in Fig. Now the velocity diagram, as shown in Fig.is drawn as discussed below:

1. Since O and A are fixed points, therefore these points are marked as one point in velocity diagram. Now draw vector ab in a direction perpendicular to AB , to some suitable scale, to represent the velocity of slider B with respect to A i.e.V_{BA}, such that

vector ab = V_{BA} = 1.9 m/s

2. From point o, draw vector ob' perpendicular to OB' to represent the velocity of coincident point B' (on the link OC) with respect to O i.e. $v_{B'O}$ and from point b draw vector bb' parallel to the path of motion of B' (which is along the link OC) to represent the velocity of coincident point B' with respect to the slider B i.e. $v_{B'B}$. The vectors ob' and bb' intersect at *b'*.

3. Since the point C lies on OB' produced, therefore, divide vector ob' at c in the same ratio as C divides OB' in the space diagram. In other words, ob′ / oc = OB′ /OC

The vector oc represents the velocity of C with respect to O i.e. V_{CO}.

4. Now from point c, draw vector cd perpendicular to CD to represent the velocity of D with respect to C i.e. V_{DC} ,and from point o draw vector od parallel to the path of motion of D (which is along the horizontal) to represent the velocity of D i.e. V_D.The vectors cd and od intersect at d.

By measurement, we find that velocity of the ram D,

V_D = vector od = 2.15 m/s Ans.

From velocity diagram, we also find that

Velocity of B with respect to B',

V_{BB}' = vector b'b = 1.05 m/s

Velocity of D with respect to C,

V_{DC} = vector cd = 0.45 m/s

Velocity of B' with respect to O

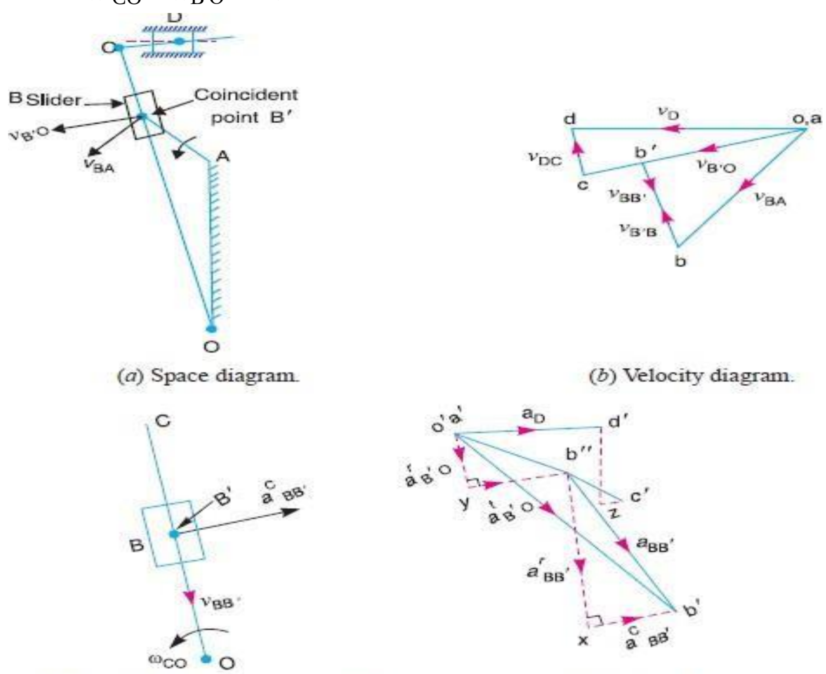
$V_{B'O}$ = vector ob' = 1.55 m/s

Velocity of C with respect to O,

V_{CO} = vector oc = 2.15 m/s

Angular velocity of the link OC or OB',

$\omega_{CO} = \omega_{B'O}$ = 3.07 rad/s



(a) Space diagram.

(b) Velocity diagram.

(c) Direction of coriolis component.

(d) Acceleration diagram.

Acceleration of the ram D

We know that radial component of the acceleration of B with respect to A,

$a^r_{BA} = \omega_{CO}$ *AB=23.7 m/s^2

Coriolis component of the acceleration of slider B with respect to the coincident point B',

a^c_{BB} =2 ω v = 6.45 m/s^2 ...(_ ω = ωCO and v = vBB′)

Radial component of the acceleration of D with respect to C,

$a^r_{DC} = v^2$DC/CD=1.01 m/s2

Radial component of the acceleration of the coincident point B' with respect to O,

$a^r_{B'O}$=4.62 m/s^2...(By measurement B'O = 0.52 m)

Now the acceleration diagram, as shown in Fig, is drawn as discussed below:

. 1. Since O and A are fixed points, therefore these points are marked as one point in the acceleration diagram. Draw vector a'b' parallel to AB, to some suitable scale, to represent the radial component of the acceleration of B with respect to A i.e. ar BA such that

vector $a'b' = a^r{}_{BA} = 23.7$ m/s^2

2. The acceleration of the slider B with respect to the coincident point B' has the following two components :

(i) Coriolis component of the acceleration of B with respect to B' i.e. $a^c{}_{BB'}$, and

(ii) Radial component of the acceleration of B with respect to B' i.e. $a^r{}_{BB'}$.

These two components are mutually perpendicular. Therefore from point b' draw vector b'x perpendicular to B'O i.e. in a direction as shown in Fig. to represent $a^c{}_{BB'}$ = 6.45 m/s2. The direction of BB ac ′ is obtained by rotating $V_{BB'}$ (represented by vector b'b in velocity diagram) through 90° in the same sense as that of link OC which rotates in the counter clockwise direction. Now from point x, draw vector xb" perpendicular to vector b'x (or parallel to B'O) to represent $a^r{}_{BB'}$ whose magnitude is yet unknown.

3. The acceleration of the coincident point B' with respect to O has also the following twocomponents:

(i) Radial component of the acceleration of coincident point B' with respect to O i.e. $a^r{}_{B'O}$

(ii) Tangential component of the acceleration of coincident point B' with respect to O, i.e. $a^t{}_{B'O}$.

These two components are mutually perpendicular. Therefore from point o', draw vector o'y parallel to B'O to represent $a^r{}_{B'O}$=4.62 m/s^2 and from point y draw vector yb" perpendicular to vector o'y to represent. at ′ The vectors xb" and yb" intersect at b". Join o'b". The vector o'b" represents the acceleration of B' with respect to O, i.e. $a_{B'O}$.

4. Since the point C lies on OB' produced, therefore divide vector o'b" at c' in the same ratio as C divides OB' in the space diagram. In other words,

o'b"/o'c' = OB'/OC

5. The acceleration of the ram D with respect to C has also the following two components:

(i) Radial component of the acceleration of D with respect to C i.e. $a^r{}_{DC}$, and

(ii) Tangential component of the acceleration of D with respect to C, i.e. $a^t{}_{DC}$.

The two components are mutually perpendicular. Therefore draw vector c'z parallel to CD to represent $a^r{}_{DC}$= 1.01 m/s^2 and from z draw zd' perpendicular to vector zc' to represent DC, at whose magnitude is yet unknown.

6. From point o', draw vector o'd' in the direction of motion of the ram D which is along the

horizontal. The vectors zd' and o'd' intersect at d'. The vector o'd' represents the acceleration of ram D i.e. a_D.

By measurement, we find that acceleration of the ram D,

$$a_D = \text{vector } o'd' = 8.4 \text{ m/s2 Ans.}$$

Angular acceleration of the slotted lever

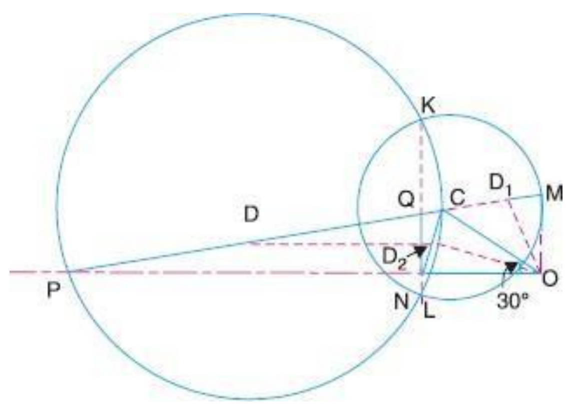
By measurement from acceleration diagram, we find that tangential component of the coincident point B' with respect to O, $a^t_{B'O=}$ vector yb''= 6.4 m/s^2

We know that angular acceleration of the slotted lever,

$$a^t_{B'O}/OB'=12.3 \text{ rad/s}^2$$

Example 3. The crank and connecting rod of a reciprocating engine are 200 mm and 700 mm respectively. The crank is rotating in clockwise direction at 120 rad/s. Find with the help of Klein's construction: 1. Velocity and acceleration of the piston, 2. Velocity and acceleration of the mid point of the connecting rod, and 3. Angular velocity and angular acceleration of the connecting rod, at the instant when the crank is at 30° to I.D.C. (inner dead centre).

Solution. Given: OC = 200 mm = 0.2 m ; PC = 700 mm = 0.7 m ; ω = 120 rad/s



The Klein's velocity diagram OCM and Klein's acceleration diagram CQNO as shown in Fig.is drawn to some suitable scale. By measurement, we find that

OM = 127mm = 0.127 m ; CM = 173 mm = 0.173 m ; QN = 93 mm = 0.093 m ; NO = 200 mm= 0.2 m

1. Velocity and acceleration of the piston

We know that the velocity of the piston P,

$$v_P = \omega\times OM = 120 \times 0.127 = 15.24 \text{ m/s}$$

and acceleration of the piston P,

$$a_P = \omega^2\times NO = (120)2 \times 0.2 = 2880 \text{ m/s2}$$

2. Velocity and acceleration of the mid-point of the connecting rod

In order to find the velocity of the mid-point D of the connecting rod, divide CM at D1 in the same ratio as D divides CP. Since D is the mid-point of CP, therefore D1 is the mid-point of CM, i.e. CD1 = D1M. Join OD1. By measurement

$$OD1 = 140 \text{ mm} = 0.14 \text{ m}$$

$$\text{Velocity of D, } V_D = \omega \times OD1 = 120 \times 0.14 = 16.8 \text{ m/s}$$

In order to find the acceleration of the mid-point of the connecting rod, draw a line DD_2 parallel to the line of stroke PO which intersects CN at D_2. By measurement

$$OD_2 = 193 \text{ mm} = 0.193 \text{ m}$$

Acceleration of D,

$$a_D = \omega^2 \times OD_2 = (120)2 \times 0.193 = 2779.2 \text{ m/s}^2$$

3. Angular velocity and angular acceleration of the connecting rod

We know that the velocity of the connecting rod *PC* (*i.e.* velocity of *P* with respect to *C*)

$$V_{PC} = \omega \times CM = 120 \times 0.173 = 20.76 \text{ m/s}$$

∴ Angular acceleration of the connecting rod *PC*,

$$\omega_{PC} = \frac{v_{PC}}{PC} = \frac{20.76}{0.7} = 29.66 \text{ rad/s} \quad \textbf{Ans.}$$

We know that the tangential component of the acceleration of *P* with respect to *C*,

$$a^t_{PC} = \omega^2 \times QN = (120)^2 \times 0.093 = 1339.2 \text{ m/s}^2$$

∴ Angular acceleration of the connecting rod *PC*,

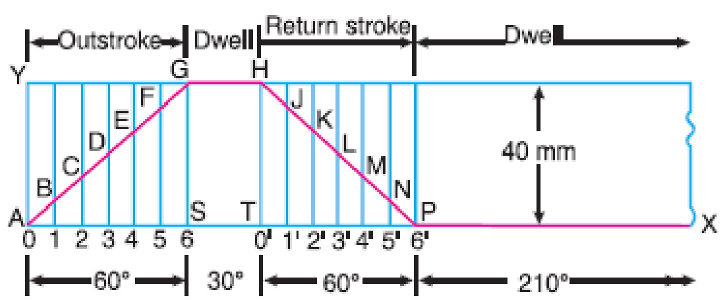
$$\alpha_{PC} = \frac{a^t_{PC}}{PC} = \frac{1339.2}{0.7} = 1913.14 \text{ rad/s}^2 \quad \textbf{Ans.}$$

Experiment No-05

Solution of minimum three (including Graphical and Analytical) problem on cams

Example 1. A cam is to give the following motion to a knife-edged follower :1. Outstroke during 60° of cam rotation ; 2. Dwell for the next 30° of cam rotation ; 3. Return stroke during next 60° of cam rotation, and 4. Dwell for the remaining 210° of cam rotation. The stroke of the follower is 40 mm and the minimum radius of the cam is 50 mm. The follower moves with uniform velocity during both the outstroke and return strokes. Draw the profile of the cam when
(a) the axis of the follower passes through the axis of the cam shaft, and
(b) the axis of the follower is offset by 20 mm from the axis of the cam shaft.

Solution following steps :
1. Draw a horizontal line AX = 360° to some suitable scale. On this line, mark AS = 60° to represent outstroke of the follower, ST = 30° to represent dwell, TP = 60° to represent return stroke and PX = 210° to represent dwell.
2. Draw vertical line AY equal to the stroke of the follower (i.e. 40 mm) and complete the rectangle as shown in Fig. 20.10.
3. Divide the angular displacement during outstroke and return stroke into any equal number of even parts (say six) and draw vertical lines through each point.
4. Since the follower moves with uniform velocity during outstroke and return stroke, therefore the displacement diagram consists of straight lines. Join AG and HP.
5. The complete displacement diagram is shown by AGHPX .



(a) Profile of the cam when the axis of follower passes through the axis of cam shaft

The profile of the cam when the axis of the follower passes through the axis of the cam shaft, is drawn as discussed in the following steps
1. Draw a base circle with radius equal to the minimum radius of the cam (i.e. 50 mm) with O as centre.
2. Since the axis of the follower passes through the axis of the cam shaft, therefore mark trace point A, .
3. From OA, mark angle AOS = 60° to represent outstroke, angle SOT = 30° to represent

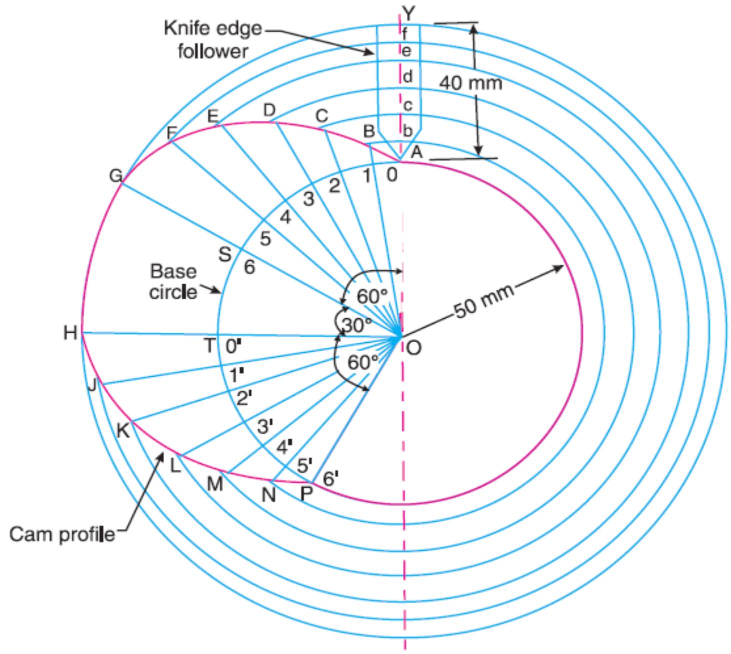
dwell and angle TOP = 60° to represent return stroke.

4. Divide the angular displacements during outstroke and return stroke (i.e. angle AOS and angle TOP) into the same number of equal even parts as in displacement diagram.

5. Join the points 1, 2, 3 ...etc. and 0′ ,1′ , 2′ , 3′ , ... etc. with centre O and produce beyond the base circle as shown in Fig. 20.11.

6. Now set off 1B, 2C, 3D ... etc. and 0′ H,1′ J ... etc. from the displacement diagram.

7. Join the points A, B, C,... M, N, P with a smooth curve. The curve AGHPA is the complete profile of the cam.



(b) Profile of the cam when the axis of the follower is offset by 20 mm from the axis of the cam shaft

The profile of the cam when the axis of the follower is offset from the axis of the cam shaft, is drawn as discussed in the following steps :

1. Draw a base circle with radius equal to the minimum radius of the cam (i.e. 50 mm) with O as centre.

2. Draw the axis of the follower at a distance of 20 mm from the axis of the cam, which intersects the base circle at A.

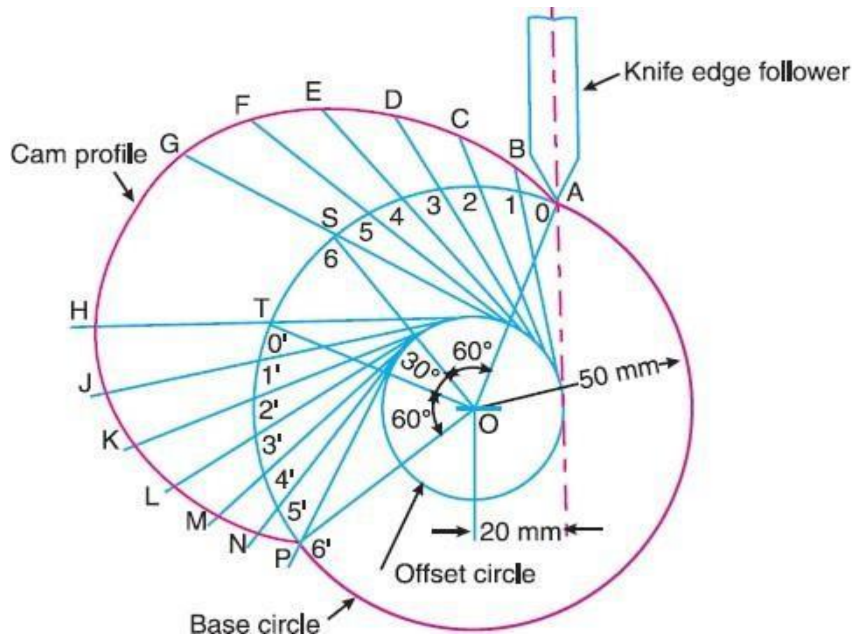
3. Join AO and draw an offset circle of radius 20 mm with centre O.

4. From OA, mark angle AOS = 60° to represent outstroke, angle SOT = 30° to represent dwell and angle TOP = 60° to represent return stroke.

5. Divide the angular displacement during outstroke and return stroke (i.e. angle AOS and angle TOP) into the same number of equal even parts as in displacement diagram.

6. Now from the points 1, 2, 3 ... etc. and 0′,1′, 2′,3′ ... etc. on the base circle, draw tangents to the offset circle and produce these tangents beyond the base circle .

7. Now set off 1B, 2C, 3D ... etc. and 0′ H,1′ J ... etc. from the displacement diagram.
8. Join the points A, B, C ...M, N, P with a smooth curve. The curve AGHPA is the complete profile of the cam.



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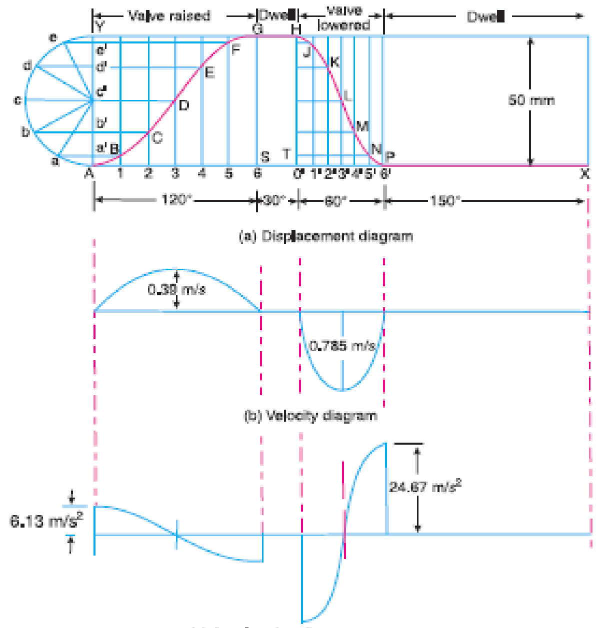
Example 2. A cam, with a minimum radius of 25 mm, rotating clockwise at a uniform speed is to be designed to give a roller follower, at the end of a valve rod, motion described below :
1. To raise the valve through 50 mm during 120° rotation of the cam ;
2. To keep the valve fully raised through next 30°;
3. To lower the valve during next 60°; and
4. To keep the valve closed during rest of the revolution i.e. 150° ;
The diameter of the roller is 20 mm and the diameter of the cam shaft is 25 mm.
Draw the profile of the cam when (a) the line of stroke of the valve rod passes through the axis of the cam shaft, and (b) the line of the stroke is offset 15 mm from the axis of the cam shaft. The displacement of the valve, while being raised and lowered, is to take place with simple harmonic motion. Determine the maximum acceleration of the valve rod when the cam shaft rotates at 100 r.p.m. Draw the displacement, the velocity and the acceleration diagrams for one complete revolution of the cam

Solution: Given : S = 50 mm = 0.05 m ; $\theta O = 120° = 2\pi/3$ rad = 2.1 rad ;
$\theta R = 60° = \pi/3$ rad = 1.047 rad ; N = 100 r.p.m.
Since the valve is being raised and lowered with simple harmonic motion, therefore the displacement diagram, as shown in Fig.



(a) Displacement diagram

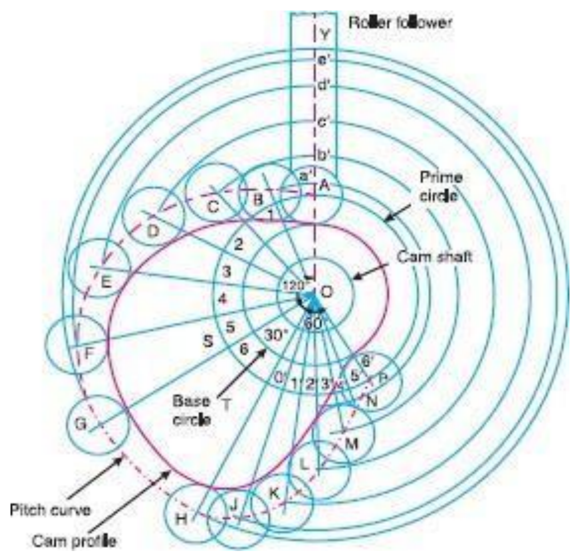
(b) Velocity diagram

(c) Acceleration diagram

(a) Profile of the cam when the line of stroke of the valve rod passes through the axis of the cam shaft

The profile of the cam, as shown in Fig. 20.17, is drawn as discussed in the following steps :

1. Draw a base circle with centre O and radius equal to the minimum radius of the cam (i.e. 25 mm).



2. Draw a prime circle with centre O and radius,

OA = Min. radius of cam +1/2 Dia. of roller =25+1/2*20=35mm

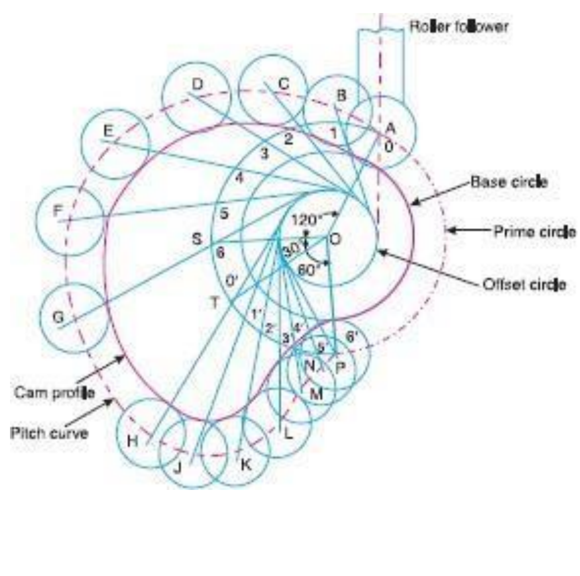
3. Draw angle AOS = 120° to represent raising or out stroke of the valve, angle SOT = 30° to represent dwell and angle TOP = 60° to represent lowering or return stroke of the valve.

4. Divide the angular displacements of the cam during raising and lowering of the valve (i.e. angle AOS and TOP) into the same number of equal even parts as in displacement diagram.

5. Join the points 1, 2, 3, etc. with the centre O and produce the lines beyond prime circle as shown in Fig. 20.17.
6. Set off 1B, 2C, 3D etc. equal to the displacements from displacement diagram.
7. Join the points A, B, C ... N, P, A. The curve drawn through these points is known as pitch curve.
8. From the points A, B, C ... N, P, draw circles of radius equal to the radius of the roller.
9. Join the bottoms of the circles with a smooth curve as shown in Fig. 20.17. This is the required profile of the cam.

(b) Profile of the cam when the line of stroke is offset 15 mm from the axis of the cam shaft

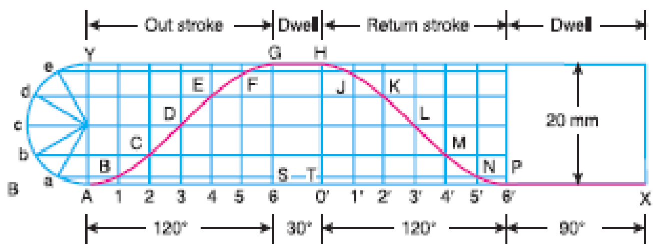
The profile of the cam when the line of stroke is offset from the axis of the cam shaft, as shown in Fig. 20.18, may be drawn as discussed in the following steps :
1. Draw a base circle with centre O and radius equal to 25 mm.
2. Draw a prime circle with centre O and radius OA = 35 mm.
3. Draw an off-set circle with centre O and radius equal to 15 mm.
4. Join OA. From OA draw the angular displacements of cam i.e. draw angle AOS = 120°, angle SOT = 30° and angle TOP = 60°.
5. Divide the angular displacements of the cam during raising and lowering of the valve into the same number of equal even parts (i.e. six parts) as in displacement diagram.
6. From points 1, 2, 3 .... etc. and 0′ ,1′ , 3′ , ...etc. on the prime circle, draw tangents to the offset circle.
7. Set off 1B, 2C, 3D... etc. equal to displacements as measured from displacement diagram.
8. By joining the points A, B, C ... M, N, P, with a smooth curve, we get a pitch curve.
9. Now A, B, C...etc. as centre, draw circles with radius equal to the radius of roller.
10. Join the bottoms of the circles with a smooth curve as shown in Fig. 20.18. This is the required profile of the cam.



Example 3. A cam drives a flat reciprocating follower in the following manner :During first 120° rotation of the cam, follower moves outwards through a distance of 20 mm with simple harmonic motion. The follower dwells during next 30° of cam rotation. During next120° of cam rotation, the follower moves inwards with simple harmonic motion. The follower dwells for the next 90° of cam rotation. The minimum radius of the cam is 25 mm. Draw the profile of the cam.

Construction

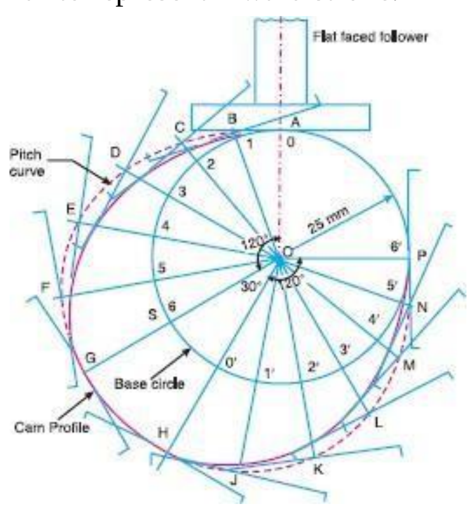
Since the follower moves outwards and inwards with simple harmonic motion, therefore the displacement diagram, as shown in fig.



Now the profile of the cam driving a flat reciprocating follower, as discussed in the following steps :

1. Draw a base circle with centre O and radius OA equal to the minimum radius of the cam (i.e. 25 mm).

2. Draw angle AOS = 120° to represent the outward stroke, angle SOT = 30° to represent dwell and angle TOP = 120° to represent inward stroke.



3. Divide the angular displacement during outward stroke and inward stroke (i.e. angles AOS and TOP) into the same number of equal even parts as in the displacement diagram.

5. From points 1, 2, 3 . . . etc., set off 1B, 2C, 3D . . . etc. equal to the distances measured from the displacement diagram.

6. Now at points B, C, D . . . M, N, P, draw the position of the flat-faced follower. The axis of the follower at all these positions passes through the cam centre.

7. The curve drawn tangentially to the flat side of the follower is the required profile of the Cam

Experiment No 6
Study of Dynamometer

Introduction:

A dynamometer is a brake but in addition it has a device to measure the frictional resistance. Knowing the frictional resistance, we may obtain the torque transmitted and hence the power of the engine.

Types of Dynamometers :

Following are the two types of dynamometers, used for measuring the brake power of an engine.

1. Absorption dynamometers, and 2. Transmission dynamometers.

In the absorption dynamometers, the entire energy or power produced by the engine is absorbed by the friction resistances of the brake and is transformed into heat, during the process of measurement. But in the transmission dynamometers, the energy is not wasted in friction but is used for doing work. The energy or power produced by the engine is transmitted through the dynamometer to some other machines where the power developed is suitably measured.

Classification of Absorption Dynamometers:

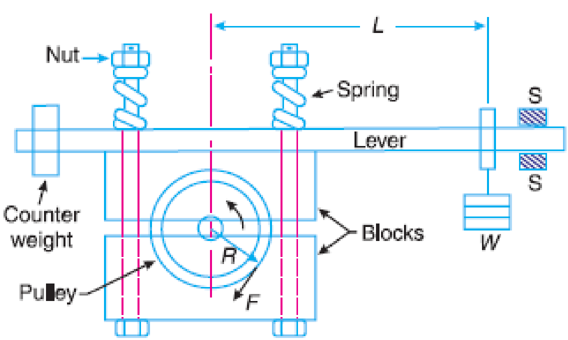
The following two types of absorption dynamometers are important from the subject point of view:

1. Prony brake dynamometer, and 2. Rope brake dynamometer.

These dynamometers are discussed, in detail, in the following pages.

1. Prony Brake Dynamometer

A simplest form of an absorption type dynamometer is a prony brake dynamometer, as shown in Fig. It consists of two wooden blocks placed around a pulley fixed to the shaft of an engine whose power is required to be measured. The blocks are clamped by means of two bolts and nuts, as shown in Fig. A helical spring is provided between the nut and the upper block to adjust the pressure on the pulley to control its speed. The upper block has a long lever attached to it and carriesa weight W at its outer end. A counter weight is placed at the other end of the lever which balances the brake when unloaded. Two stops S, S are provided to limit the motion of the lever.



When the brake is to be put in operation, the long end of the lever is loaded with suitable weights W and the nuts are tightened until the engine shaft runs at a constant speed and the lever is in horizontal position. Under these conditions, the moment due to the weight W must balance the moment of the frictional resistance between the blocks and the pulley.

W = Weight at the outer end of the lever in newtons,

L = Horizontal distance of the weight W from the centre of the pulley in metres,

F = Frictional resistance between the blocks and the pulley in newtons,

R = Radius of the pulley in metres, and N = Speed of the shaft in r.p.m.

We know that the moment of the frictional resistance or torque on the shaft,

$T = W.L = F.R$ N-m

Work done in one revolution = Torque × Angle turned in radians

$= T \times 2\Pi$N-m

Work done per minute = $T \times 2\Pi N$ N-m

We know that brake power of the engine

$$B.P. = \frac{\text{Work done per min.}}{60} = \frac{T \times 2\pi N}{60} = \frac{W\,L \times 2\pi\,N}{60} \text{ watts}$$

2. Rope Brake Dynamometer

It is another form of absorption type dynamometer which is most commonly used for measuring the brake power of the engine. It consists of one, two or more ropes wound around the flywheel or rim of a pulley fixed rigidly to the shaft of an engine. The upper end of the ropes is attached to a spring balance while the lower end of the ropes is kept in position by applying a dead weight as shown in Fig.. In order to prevent the slipping of the rope over the flywheel, wooden blocks are placed at intervals around the circumference of the flywheel. In the operation of the brake, the engine is made to run at a constant speed. The frictional torque, due to the rope, must be equal to the torque being transmitted by the engine.

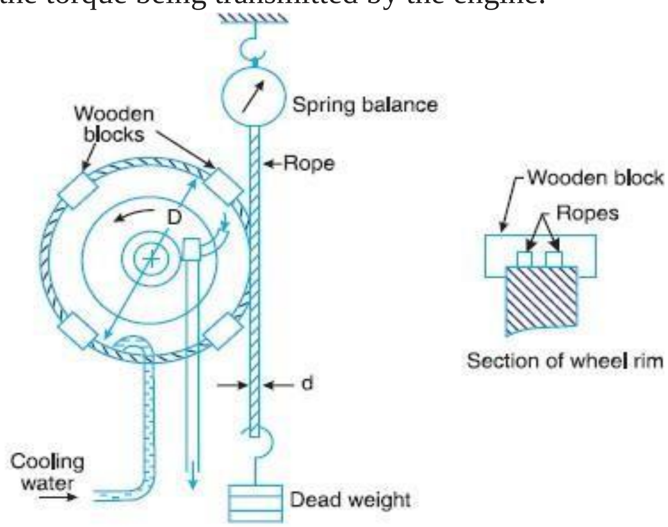


Fig. 19.32. Rope brake dynamometer.

∴ Brake power of the engine,

$$\text{B.P} = \frac{\text{Work done per min}}{60} = \frac{(W - S)\,\pi\,(D + d)N}{60} \text{ watts}$$

W = Dead load in newtons,

S = Spring balance reading in newtons,

D = Diameter of the wheel in metres,

d = diameter of rope in metres, and

N = Speed of the engine shaft in r.p.m.

Net load on the brake

$= (W – S)$ N

We know that distance moved in one revolution

$= \Pi (D+ d)$ m

Work done per revolution
= (W --) Π □ (D + d) N-m
and work done per minute
= (W - S) Π ((D + d) N N-m

If the diameter of the rope (d) is neglected, then brake power of the engine,

$$B.P. = \frac{(W - S)\,\pi D\,N}{60} \text{ watts}$$

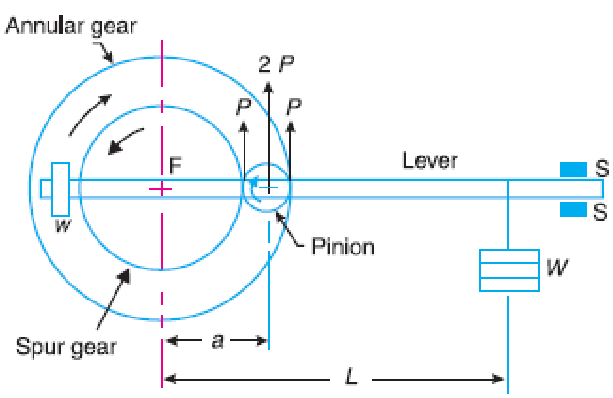
Classification of Transmission Dynamometers:
The following types of transmission dynamometers are important from the subject point of view :

1. Epicyclic-train dynamometer, 2. Belt transmission dynamometer, and 3. Torsion dynamometer.

We shall now discuss these dynamometers, in detail, in the following pages.

1. Epicyclic-train Dynamometer:

An epicyclic-train dynamometer, as shown in Fig., consists of a simple epicyclic train of gears, i.e. a spur gear, an annular gear (a gear having internal teeth) and a pinion. The spur gear is keyed to the engine shaft (i.e. driving shaft) and rotates in anticlockwise direction. The annular gear is also keyed to the driving shaft and rotates in clockwise direction. The pinion or the intermediate gear meshes with both the spur and annular gears. The pinion revolves freely on a lever which is pivoted to the common axis of the driving and driven shafts. A weight w is placed at the smaller end of the lever in order to keep it in position. A little consideration will show that if the friction of the pin on which the pinion rotates is neglected, then the tangential effort P exerted by the spur gear on the pinion and the tangential reaction of the annular gear on the pinion are equal. Since these efforts act in the upward direction as shown, therefore total upward force on the lever acting through the axis of the pinion is 2P.



This force tends to rotate the lever about its fulcrum and it is balanced by a dead weight W at the end of the lever. The stops S, S are provided to control the movement of the lever.

For equilibrium of the lever, taking moments about the fulcrum F,

$$2P \times a = W.L \text{ or } P = W.L/2a$$

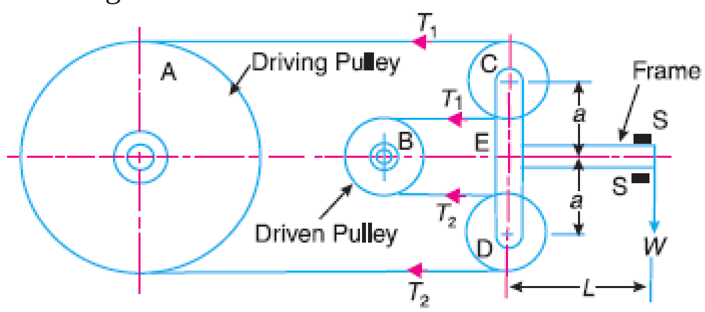
Let R = Pitch circle radius of the spur gear in metres, and
N = Speed of the engine shaft in r.p.m.

Torque transmitted, T = P.R

and power transmitted $= \dfrac{T \times 2\pi N}{60} = \dfrac{P.R \times 2\pi N}{60}$ watts

2. Belt Transmission Dynamometer-Froude or Thorneycroft Transmission Dynamometer:

When the belt is transmitting power from one pulley to another, the tangential effort on the driven pulley is equal to the difference between the tensions in the tight and slack sides of the belt. A belt dynamometer is introduced to measure directly the difference between the tensions of the belt, while it is running



A belt transmission dynamometer, as shown in Fig. is called a Froude or Throneycroft transmission dynamometer. It consists of a pulley A (called driving pulley) which is rigidly fixed to the shaft of an engine whose power is required to be measured. There is another pulley B (called driven pulley) mounted on another shaft to which the power from pulley A is transmitted. The pulleys A and B are connected by means of a continuous belt passing round the two loose pulleys C and D which are mounted on a T-shaped frame. The frame is pivoted at E and its movement is controlled by two stops S,S. Since the tension in the tight side of the belt (T1) is greater than the tension in the slack side of the belt (T2), therefore the total force acting on the pulley C (i.e. 2T1) is greater than the total force acting on the pulley D (i.e. 2T2). It is thus obvious that the frame causes movement about E in the anticlockwise direction. In order to balance it, a weight W is applied at a distance L from E on the frame as shown in fig.

Now taking moments about the pivot E, neglecting friction,

$$2T_1 \times a = 2T_2 \times a + W.L \quad \text{or} \quad T_1 - T_2 = \frac{W.L}{2a}$$

Let D = diameter of the pulley A in metres, and
N = Speed of the engine shaft in r.p.m.

$\therefore$ Work done in one revolution $= (T_1 - T_2)\,\pi\, D$ N-m

and workdone per minute $= (T_1 - T_2)\,\pi\, DN$ N-m

$\therefore$ Brake power of the engine, $\text{B.P.} = \dfrac{(T_1 - T_2)\,\pi\, DN}{60}$ watts

3. Torsion Dynamometer:

A torsion dynamometer is used for measuring large powers particularly the power transmitted along the propeller shaft of a turbine or motor vessel. A little consideration will show that when the power is being transmitted, then the driving end of the shaft twists through a small angle relative to the driven end of the shaft. The amount of twist depends upon many factors such as torque acting on the shaft (T), length of the shaft (l), diameter of the shaft (D) and modulus of rigidity (C) of the material of the shaft. We know that the torsion equation is

$$\frac{T}{J} = \frac{C.\theta}{l}$$

where θ = Angle of twist in radians, and

J = Polar moment of inertia of the shaft.

For a solid shaft of diameter D, the polar moment of inertia

$$J = \frac{\pi}{32} \times D^4$$

and for a hollow shaft of external diameter D and internal diameter d, the polar moment of inertia,

$$J = \frac{\pi}{32}(D^4 - d^4)$$

From the above torsion equation,

$$T = \frac{C.J}{l} \times \theta = k.\theta$$

where $k = C.J/l$ is a constant for a particular shaft. Thus, the torque acting on the shaft is proportional to the angle of twist. This means that if the angle of twist is measured by some means, then the torque and hence the power transmitted may be determined.

We know that the power transmitted

$$P = \frac{T \times 2\pi N}{60}$$ watts, where N is the speed in r.p.m.

Experiment No. 07

To study various kinds of Belt drives.

Theory:

The belts or ropes are used to transmit power from one shaft to another by means of pulleys, which rotate at the same speed or at different speeds. The amount of power transmitted depends upon the following factors:

1. The velocity of belt.
2. The tension under which the belt is placed on the pulleys.
3. The arc of contact between the belt and the smaller pulley.
4. The conditions under which the belt is used.

Selection of a Belt Drive

Following are the various important factors upon which the selection of a belt depends:
1. Speed of the driving and driven shaft. 2. Speed reduction ratio,
3. Power to be transmitted, 4. Centre distance between the shafts,
5. Positive drive requirements, 6. Shafts layout,
7. Space available, and 7. Service conditions.

Types of Belt Drives:

The belt drives are usually classified into the following three groups:

1. **Light drives:** These are used to transmit small powers at belt speeds upto about 10m/s, as in agricultural machines and small machine tools.
2. **Medium drives:** These are used to transmit medium power at belt speeds over 10m/s but up to 22m/s, as machine tools.
3. **Heavy drives:** These are used to transmit large powers at belt speeds above 22 m/s, as in compressors and generators.

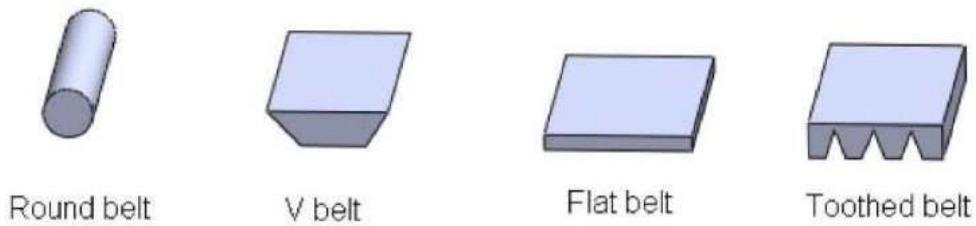
Types of Belts:

Though there are many types of belts used these days, yet the following are important from the subject point of view:

1. Flat belt: - The flat belt is mostly used in the factories and workshops, where a moderate amount of power is to be transmitted, from one pulley to another, when the two pulleys are not more than 8 meters apart.

2. V-Belt: - The V-belt is mostly used in the factories and workshop, where a moderate amount of power is to be transmitted, from one pulley to another, when the two pulleys are very near to each other.

3. Circular belt or rope: - The circular belt or rope, is mostly used in the factories and workshops, where a great amount of power is to be transmitted, from one pulley to another, when the two pulleys are more than 8 meters apart.



Materials used for belts

The material used for belts and ropes must be strong, flexible, and durable. It must have a high coefficient of friction. The belts, according to the material used, are classified as follows:

1. Leather belts: The most important materials for the belt is leather. The best leather belts are made from 1.2 metres to 1.5 metres long strips cut from either side of the back bone of the top grade steer hides. The hair side of the lather is smoother and harder than the flesh side, but the flesh side is stronger. The fibers on the hair side are perpendicular to the surface, while those on the flesh side are interwoven and parallel to the surface.

Therefore for these reasons, the hair side of a belt should be in contact with the pulley surface. This gives a more intimate contact between the belt and the pulley and places the greatest tensile strength of the belt section on the outside, where the tension is maximum as the belt over the pulley.

2. Cotton or a fabric belts: Most of the belts are made by folding canvass or cotton duck to three or more layers (depending upon the thickness desired) and stitching together. These belts are woven also into a strip of the desired width and thickness. They are impregnated with some filler like linseed oil in order to make the belts waterproof and to prevent injury to the fibers. The cotton belts are cheaper and suitable in warm climates, in damp atmospheres and in exposed positions. Since the cotton belts require little attention, therefore these belts are mostly used in farm machinery, belt conveyor etc.

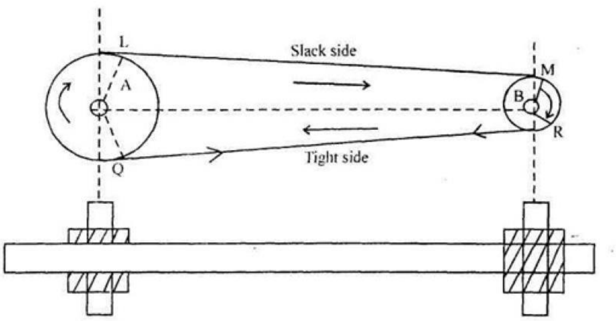
3. Rubber belt: The rubber belts are made of layers of fabric impregnated with rubber composition and have a thin layer of rubber on the faces. These belts are very flexible but are quickly destroyed if allowed to come into contact with heat, oil or grease. One of the principle advantages of these belts is that they may be easily made endless. These belts are found suitable for sawmills; pare mills where they are exposed to moisture.

4. Balata belts: These belts are similar to rubber belts except that balata gum is used in place of rubber. These belts are acid proof and waterproof and it is not affected by animal oils or alkalies. The balata belts should not be at temperatures above 400 0C because at this temperature the balata begins to soften and becomes sticky. The strength of balata belts is 25 percent higher than rubber belts.

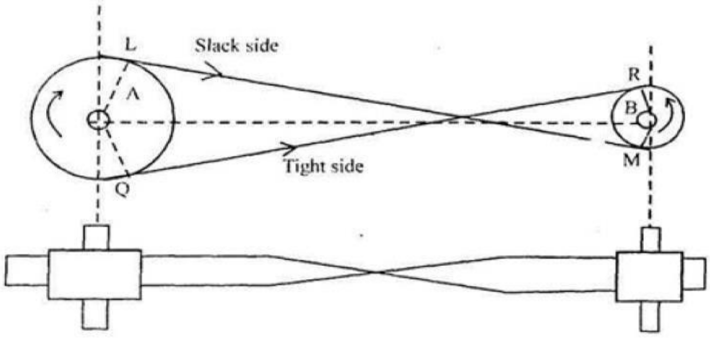
Types of Flat Belt Drives

The power from one pulley to another may be transmitted by any of the following types of belt drives.

1. Open belt drive: The open belt drive is used with shafts arranged parallel and rotating in the same direction. In this case, the driver A pulls the belt from one side (i.e. lower side RQ) and\ delivers it to the other side (i.e. upper side LM). Thus the tension in the lower side belt will be more than that in the upper side belt. The lower side belt (because of more tension) is known as tight side whereas the upper side belt (because of less tension) is known as slack side.



2. Crossed or twist belt drive: The crossed or twist belt drive is used with shafts arranged parallel and rotating in the opposite directions. In this case, the driver pulls the belts from one side (i.e. RQ) and delivers it to the other side (i.e. LM). Thus the tension in the belt RQ will be more that that in the belt LM. The belt RQ (because of more tension) is known as tight side, where as belt LM (because of less tension) is known as slack side.



3. Quarter turn belt drive: The quarters turn belt drive also known as right angle belt drive is used with shafts arranged at an angle and rotating in one definite direction. In order to prevent the belt from leaving the pulley, the width of the face of the face of the pulley should be greater or equal to 1.4 b, where b is the width of belt. In case the pulleys cannot be arranged, when the reversible motion is desired, then a quarter turn belt drive with guide pulley may be used.

4. Belt drive with idle pulleys: A belt drive with an idler pulley, used with shafts arranged parallel and when an open belt drive cannot be used due to small angle of contact on the smaller pulley. This type of drive is provided to obtain high velocity ratio and when the required belt tension cannot be obtained by the other means. When it is desired to transmit motion from one shaft to several shaft, all arranged in parallel, a belt drive with many idler pulleys, may be employed.

5. Compound belt drive: A compound belt drive, is used when power is transmitted from one shafts to another through a number of pulleys.

6. Stepped or cone pulley drive: A stepped or cone pulley drive, is used for changing the speed of the driven shaft while the main or driving shafts runs at constant speed. This is accomplished by shifting the belt from one part of the steps to the other.

7. Fast and loose pulley drive: A fast and loose pulley drive, is used when the driven or machine shaft is to be started or stopped when ever desired without interfering with the driving shaft. A pulley which is keyed to the machine shaft is called fast pulley and runs at the same speed as that of machine shaft. A loose pulley runs freely over the machine shaft and is incapable of transmitting any power. When the driven shaft is required to be stopped, the belt is pushed on the loose pulley by means of sliding bar having belt forks.

Experiment No. 08
To study the characteristics of Hartnell Governors.

Introduction & Description:

The drive unit consists of a small electric motor connected through 'V' belt to shaft drive. Motor & main shaft are mounted on a rigid M.S. Base plate in vertical fashion. The spindle is supported in a ball bearings. The optional governor mechanism can be mounted on spindle. Precise speed control is offered by the speed control unit, & counter hole over the spindle shaft allows the use of a tachometer to determine the speed. A graduated scale is fixed to the sleeve & guided in vertical direction. The center sleeve of the porter & Proell governor incorporates a weight sleeve to which weights may be added. The Hartnell governor provides means of varying spring rate & initial compression level & mass of rotating weight. This enables the Hartnell governor, to be operated as a stable or unstable governor.

The apparatus is designed to exhibit the characteristics of the spring loaded governor & dead weight governor. The apparatus is driven by D.C. Motor with variable speed control unit.

The apparatus can perform following experiments on

1. Watt type Governor.
2. Porter type Governor.
3. Proell type Governor.
4. Hartnell type Governor.

SPECIFICATIONS:

* Drive D.C. Motor H.P. ¼ Speed 1500 RPM

* Speed variation arrangement provided.

Separate linkage for governor arrangements mentioned above are provided using same motor & base. Speed Measurement is to be done from control unit display; sleeve displacement is to be noted on scale provided. Variable control unit is provided with the apparatus. Following experiments can be conducted on the Gravity controlled governor apparatus i.e. for watt governor, Porter Governor, Proell Governor & also on spring loaded governor

EXPERIMENTAL PROCEDURE:

The governor mechanism under test is fitted with the chosen rotating weights & spring, where applicable & inserted into the drive unit. The following simple procedure may then be followed.

The control unit is switched on & the speed control slowly rotated, increasing the governor speed until the center sleeve rises off the lower stop & aligns with the first division on the graduated scale. The sleeve position & speed are then recorded. Speed may be determined from control unit. The governor speed is then increased in steps to give suitable sleeve movements, & readings repeated at each stage throughout the range of the sleeve movement possible.

The result may be plotted as curves of speed against sleeve position.
Further tests are carried out changing the value of variable at a time to draw curves.

OPERATING INSTRUCTIONS:

For obtaining the graphs as mentioned above following instructions may be followed.

1. Arrange the setup as a Watt/Porter, Proell governor or Hartnell. This canbe done by removing the upper sleeve on the vertical spindle of the governor & using proper linkages provided.
2. Make proper connections of the motor.
3. Increase the motor speed gradually.
4. Note the sleeve displacement on the scale provided & speed
5. Calculate the height of governor, radius of governor & centrifugal force.

PRECAUTIONS:

1. Do not keep the mains 'ON' when trial is complete.
2. Increase the speed gradually. Take the sleeve displacement readingwhen the pointer remains steady.
3. See that at higher speed the load on sleeve dose not hit the upper sleeve ifthe governor.
4. While closing the test the dimmer to zero position & then switch 'OFF'the motor.

OBSERVATIONS :

Initial radius of Governor	(R_O)	:	mm
Weight of Ball	(m)	:	kg
Length of Vertical Link	(a)	:	mm
Length of Horizontal Link	(b)	:	mm

OBSERVATION TABLE :

Sr. No.	Speed N (rpm)	Sleeve Displacement X (mm)

CALCULATIONS :

1. Angular Velocity of Governor (ω): = $(2 \pi N) / 60$…..................... Rad/Sec
2. Radius Of Rotation (R): $R_0 + (X \times a / b)$
3. Centrifugal Force (F): $= m R \omega^2$

Result Table:

Sr. No.	Angular Velocity	Radius of Rotation (R) mm	Centrifugal Force

Conclusion:

Experiment No: 09

Verification of Ratio of angular velocities of shafts connected by hooks joint

Introduction

If we look at the history of the universal joint, Robert Hooke is commonly knownas the inventor of the 'Hooke's Joint' or 'Universal Joint'.

A universal joint (universal coupling, U-joint, Cardan joint, Spicer or Hardy Spicer joint, or Hooke's joint) is a joint or coupling connecting rigid rods whose axes are inclined to each other, and is commonly used in shafts that transmit rotary motion as shown in fig. 1. It consists of a pair of hinges located close together, oriented at 90° to each other, connected by a cross shaft. The universal joint is not a constant-velocity joint. In Europe the universal joint is often called the Cardano joint or Cardan shaft, after the Italian mathematician Gerolamo Cardano

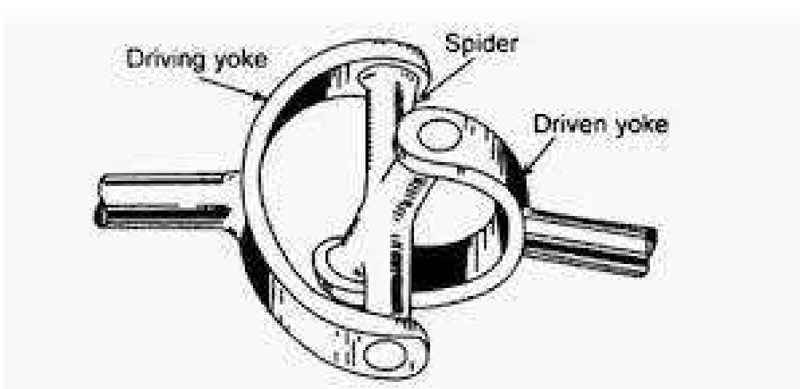


Fig. 1 simple universal joint

A Hooke’s joint is used to connect two shafts, which are intersecting at a smallangle, as shown in Fig. 2. The end of each shaft is forked to U-type and each forkprovides two bearings for the arms of a cross. The arms of the cross are perpendicular to each other as shown in fig. 2

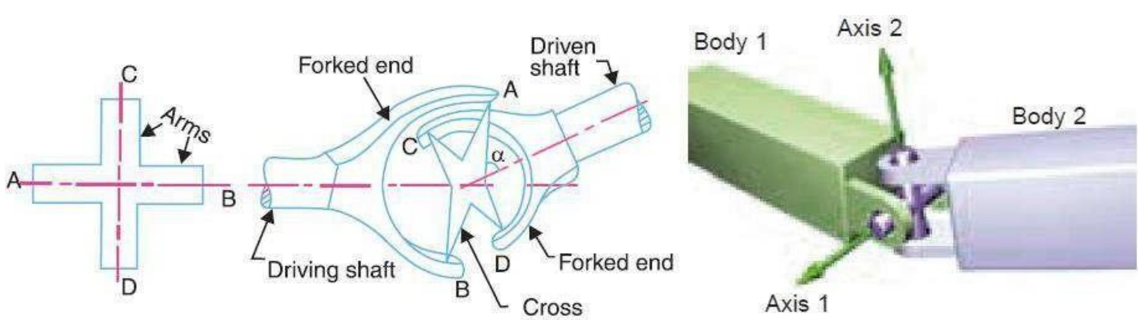


Fig. 2. Universal or Hooke’s joint.

The motion is transmitted from the driving shaft to driven shaft through a cross. The inclination of the two shafts may be constant, but in actual practice it varies, when the motion is transmitted. The main application of the Universal or Hooke’s joint is found in the transmission from the gear box to the differential or back axle of the automobiles. It is also used for transmission of power to different spindlesof multiple drilling machine. It is also used as a knee joint in milling machines.

Ratio of the Shafts Velocities

The top and front views connecting the two shafts by a universal joint are shown in Fig. 7. Let the initial position of the cross be such that both arms lie in the plane of the paper in front view, while the arm *AB* attached to the driving shaft lies in the plane containing the axes of the two shafts. Let the driving shaft rotates through an angle θ, so that the arm *AB* moves in a circle to a new position *A*1 *B*1 as shown in front view. A little consideration will show that the arm *CD* will also move in a circle of the same size. This circle when projected in the plane of paper appears to be an ellipse. Therefore the arm *CD* takes new position *C*1*D*1 on the ellipse, at an angle θ. But the true angle must be on the circular path. To find thetrue angle, project the point *C*1 horizontally to intersect the circle at *C*2. Therefore the angle *COC*2 (equal to φ) is the true angle turned by the driven shaft. Thus when the driving shaft turns through an angle θ, the driven shaft turns through an angle φ. It may be noted that it is not necessary that φ may be greater than θ or less than θ. At a particular point, it may be equal to θ.

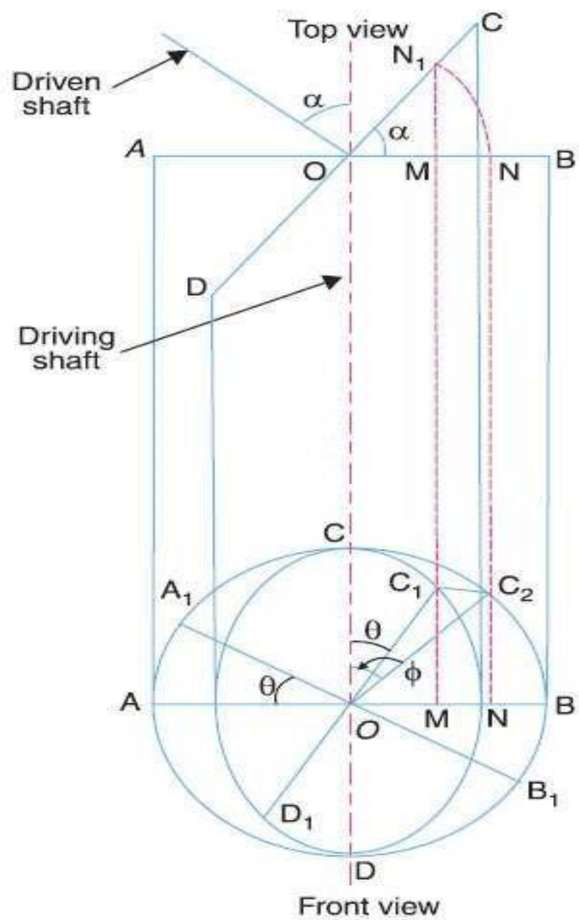


Fig. 7. *Polar diagram-salient features of driven shaft speed*

In triangle OC_1M, $\angle OC_1M = \theta$

$$\tan\theta = \frac{OM}{MC_1} \qquad \text{(i)}$$

and in triangle OC_2N, $\angle OC_2N = \varphi$

$$\tan\phi = \frac{ON}{NC_2} = \frac{ON}{MC_1}, NC2 = MC1 \qquad (ii)$$

Dividing equation (*1*) by (*2*),

$$\frac{\tan\theta}{\tan\phi} = \frac{OM}{MC_1} \cdot \frac{MC_1}{ON} = \frac{OM}{ON}$$

But $OM = ON_1 \cos\alpha = ON\cos\alpha, ON1 = ON\ (raduse\ for\ center\ o)$

$$\therefore \frac{\tan\theta}{\tan\phi} = \frac{ON\cos\alpha}{ON} \xrightarrow{yields}$$

$$\tan\theta = \tan\phi \cdot \cos\alpha \qquad (iii)$$

Let $\quad \omega$ = Angular velocity of the driving shaft = $d\theta / dt$

ω_1 = Angular velocity of the driven shaft = $d\phi / dt$

Differentiating both sides of equation (***iii***),

$$\sec^2\theta \frac{d\theta}{dt} = \cos\alpha \cdot \sec^2\phi \frac{d\phi}{dt} - \sin\alpha \cdot \tan\phi \cdot \frac{d\alpha}{dt}, (\alpha = constant \xrightarrow{yields} \frac{d\alpha}{dt} = 0)$$

$$\sec^2\theta \cdot \frac{d\theta}{dt} = \cos\alpha \cdot \sec^2\phi \cdot \frac{d\phi}{dt} \xrightarrow{yields} \sec^2\theta \cdot \omega = \cos\alpha \cdot \sec^2\phi \cdot \omega_1$$

$$\frac{\omega_1}{\omega} = \frac{\sec^2\theta}{\cos\alpha \cdot \sec^2\phi} = \frac{1}{\cos^2\theta \cos\alpha \cdot \sec^2\phi} \qquad (iv)$$

We know that:

$$\sec^2\phi = 1 + \tan^2\phi = 1 + \frac{\tan^2\theta}{\cos^2\alpha}, (from\ eq. iii)$$

$$\sec^2\phi = 1 + \frac{\sin^2\theta}{\cos^2\theta \cdot \cos^2\alpha} = \frac{\cos^2\theta \cdot \cos^2\alpha + \sin^2\theta}{\cos^2\theta \cdot \cos^2\alpha}$$

$$= \frac{\cos^2\theta \cdot (1 - \sin^2\alpha + \sin^2\theta}{\cos^2\theta \cdot \cos^2\alpha}$$

$$= \frac{\cos^2\theta - \cos^2\theta \cdot \sin^2\alpha + \sin^2\theta}{\cos^2\theta \cdot \cos^2\alpha} = \frac{1 - \cos^2\theta \cdot \sin^2\alpha}{\cos^2\theta \cdot \cos^2\alpha}$$

Substituting this value of $\sec^2\varphi$ in equation (***iv***), we have veloity ratio,

$$\frac{\omega_1}{\omega} = \frac{1}{\cos^2\theta \cos\alpha \cdot} \cdot \frac{\cos^2\theta \cdot \cos^2\alpha}{1 - \cos^2\theta \cdot \sin^2\alpha} = \frac{\cos\alpha}{1 - \cos^2\theta \cdot \sin^2\alpha} \qquad (v)$$

Note:

If N = Speed of the driving shaft in r.p.m., and

N_1 = Speed of the driven shaft in r.p.m.

Then the equation (*v*) may also be written as

$$\frac{N_1}{N} = \frac{cos\alpha}{1 - cos^2\theta \cdot sin^2\alpha}$$

3.1. Maximum and Minimum Speeds of Driven Shaft

We have discussed in the previous article that velocity ratio,

$$\frac{\omega_1}{\omega} = \frac{cos\alpha}{1 - cos^2\theta \cdot sin^2\alpha}$$

$$\omega_1 = \frac{\omega \cdot cos\alpha}{1 - cos^2\theta \cdot sin^2\alpha} \qquad (vi)$$

The value of ω_1 will be maximum for a given value of α, if the denominator of equation (*vi*) is minimum. This will happen, when $\cos^2\theta = 1$, *i.e.* when $\theta = 0°$, $180°$, $360°$ etc.

∴ Maximum speed of the driven shaft,

$$\omega_{1max} = \frac{\omega \cdot cos\alpha}{1 - \cdot sin^2\alpha} = \frac{\omega \cdot cos\alpha}{\cdot cos^2\alpha} = \frac{\omega}{cos\,\alpha} \qquad (vii)$$

$$N_{1max} = \frac{N}{cos\,\alpha}, \text{(where } N \text{ and } N1 \text{ are in r.p.m.)}$$

Similarly, the value of ω_1 is minimum, if the denominator of equation (*vi*) is maximum. This will happen, when $(\cos^2\theta \,.\, \sin^2\alpha)$ is maximum, or $\cos^2\theta = 0$, *i.e.* when $\theta = 90°$, $270°$ etc.

∴ Minimum speed of the driven shaft,

$$\omega_{1min} = \frac{\omega \cdot cos\alpha}{1 - cos^2\theta \cdot sin^2\alpha} = \omega \cdot cos\alpha$$

$$N_{1min} = \omega \cdot cos\alpha, \text{(where } N \text{ and } N1 \text{ are in r.p.m.)}$$

3.2. Angular Acceleration of the Driven Shaft

We know that:

$$\omega_1 = \frac{\omega \cdot cos\alpha}{1 - cos^2\theta \cdot sin^2\alpha} = \omega \cdot cos\alpha \cdot (1 - cos^2\theta \cdot sin^2\alpha)^{-1}$$

Differentiating the above expression, we have the angular acceleration of the driven shaft,

$$\varepsilon = \frac{d\omega_1}{dt} = \omega \cdot cos\alpha \cdot [-1(1 - cos^2\theta \cdot sin^2\alpha)^{-2} \cdot (2 \cdot cos\theta \cdot \sin\theta \cdot \sin^2\alpha)] \cdot \frac{d\theta}{dt}$$

$$\varepsilon = \frac{-\omega^2 \cdot cos\alpha \cdot \sin 2 \cdot \theta \cdot sin^2\alpha}{(1 - cos^2\theta \cdot sin^2\alpha)^2}, (2 \cdot cos\theta \cdot sin\,\theta = sin2 \cdot \theta \; and \frac{d\theta}{dt} = \omega)$$

For ***angular acceleration to be maximum***, differentiate $d\omega_1 / dt$ with respect to θ and equate to zero. The result is approximated as:

$$\cos 2 \cdot \theta = \frac{\sin^2\alpha \cdot (2 - \cos^2 2\theta)}{2 - \sin^2\alpha}$$

Note: If the value of α is *less than* 30°, then cos 2 θ may approximately be written as:

$$\cos 2 \cdot \theta = \frac{\sin^2\alpha}{2 - \sin^2\alpha}$$

4. Maximum Fluctuation of Speed

We know that the maximum speed of the driven shaft,

$$\omega_{1max} = \frac{\omega}{cos\,\alpha}$$

and minimum speed of the driven shaft:

$$\omega_{1min} = \omega \cdot cos\alpha$$

∴ Maximum fluctuation of speed of the driven shaft:

$$q = \omega_{1max} - \omega_{1min} = \frac{\omega}{cos\,\alpha} - \omega \cdot cos\alpha = \omega \cdot (\frac{1}{cos\,\alpha} - cos\alpha)$$

$$q = \omega \cdot \left(\frac{1 - cos^2\alpha}{cos\,\alpha}\right) = \omega \cdot \left(\frac{sin^2\alpha}{cos\,\alpha}\right)$$

Since α is a small angle, therefore substituting cos α = 1, and sin α = α radians.

∴ Maximum fluctuation of speed:

$$q = \omega \cdot \alpha^2$$

Hence, ***the maximum fluctuation of speed of the driven shaft approximately varies as the square of the angle between the two shafts.***